

# On the scalar potential of two-Higgs doublet models

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## Abstract

We perform a detailed analysis of the Two-Higgs Doublet Model (2HDM) potential. At the tree-level, the potential may accommodate more than one minima, one of them being the electroweak (EW) minimum where the universe lives. The parameter space allowed after the data from the Large Hadron Collider (LHC) came in almost excludes those cases where the EW vacuum is shallower than the second minimum. We extend the analysis by including terms in the 2HDM potential that break the  $Z_2$  symmetry of the potential by dimension-4 operators and show that the conclusions remain unchanged. Furthermore, a one-loop analysis of the potential is performed for both cases, namely, where the  $Z_2$  symmetry of the potential is broken by dimension-2 or dimension-4 operators. For quantitative analysis, we show our results for the Type-II 2HDM, qualitative results remaining the same for other 2HDMs. We find that the nature of the vacua from the tree-level analysis does not change; the EW vacuum still remains deeper.

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## 1 Introduction

Two-Higgs Doublet Models (2HDM) [1, 2] are one of the most popular extensions of the Standard Model (SM), even without invoking supersymmetry, for which more than one scalar doublet is a necessary condition. In a 2HDM with two doublets  $\Phi_1$  and  $\Phi_2$ , there are five physical scalars: the two  $CP$ -even neutrals  $h$  and  $H$ , the  $CP$ -odd neutral  $A$ <sup>1</sup>, and two charged scalars  $H^\pm$ . While a generic 2HDM may contain flavor-changing neutral current (FCNC) interactions (see, *e.g.*, Ref. [4, 5, 6] for such type of 2HDMs), one usually invokes some discrete symmetry to banish such FCNC at the tree-level, based on the Glashow-Weinberg-Paschos (GWP) [7] theorem that there is no tree-level FCNC if all right-handed fermions of a given electric charge couple to only one of the doublets. It turns out that there are four 2HDMs that satisfy the GWP criterion when a discrete  $Z_2$  symmetry is applied on the Lagrangian. They are:

1. Type I, for which all fermions couple with  $\Phi_2$  and none with  $\Phi_1$ ;
2. Type II, for which up-type quarks couple to  $\Phi_2$ , down-type quarks and charged leptons couple to  $\Phi_1$ ;
3. Type Y (sometimes called Type III or Flipped), for which up-type quarks and charged leptons couple to  $\Phi_2$  and down-type quarks couple to  $\Phi_1$ ;
4. Type X (sometimes called Type IV or Lepton-specific), for which all charged leptons couple to  $\Phi_1$  and all quarks couple to  $\Phi_2$ .

Among them, Type II 2HDM has been most widely investigated [8, 9, 10, 11, 12] because the scalar sector of minimal supersymmetry is a Type II 2HDM.

The tightest constraint on any 2HDM comes from the fact that the observed scalar resonance at the Large Hadron Collider (LHC) can be identified with the SM Higgs boson with  $m_{\text{Higgs}} = 125.09 \pm 0.24$

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<sup>1</sup> The neutral scalars do not have any definite  $CP$  property if the scalar potential violates  $CP$ . For 2HDM with spontaneous  $CP$  violation, see, *e.g.*, Ref. [3].

GeV [13]. Thus, the Yukawa and gauge couplings of the 2HDM must be so aligned as to make the lighter  $CP$ -even mass eigenstate  $h$  to almost coincide with the SM Higgs boson. This is known as the *alignment limit*, and while the allowed parameter space may vary from one 2HDM to the other, the qualitative results are quite similar [14, 15, 16]. There are other constraints, like the oblique parameters or the lower limit on the  $H^\pm$  mass coming from  $b \rightarrow s\gamma$  decay rate [17, 18], but such constraints are in general not equally valid for all 2HDMs <sup>2</sup>. The theoretical constraints include, just like any other extensions of the SM, the vacuum stability, validity of the perturbative nature of the couplings, and constraints coming from the requirement of unitarity of scattering amplitudes [19].

In this paper, we will focus upon the scalar potential of the 2HDMs. This is much more complicated, even at the tree-level, compared to the SM scalar potential, if both the  $CP$ -even neutral scalars are allowed to have nonzero vacuum expectation values (VEV). As has been shown in Refs. [20, 21, 22, 23, 24], the 2HDM scalar potential, even with the softly broken  $Z_2$ -symmetry, may allow more than one minima. A similar analysis was very recently done for  $Z_2$ -breaking models [25]. In Refs. [20, 22, 23, 24], the authors considered the scalar potential of a  $Z_2$ -conserving 2HDM at the tree-level, and showed that it can allow multiple normal non-equivalent stationary points (at most two of them can be minima). However, a charge-violating or  $CP$ -violating minimum cannot coexist with a normal minimum, *i.e.*, where the  $CP$ -even neutral fields get the VEV. This can be put in a more succinct way: minima with different natures cannot co-exist in 2HDM [20, 24]. Also, the data from LHC all but rules out those points where the second minimum is deeper than the electroweak (EW) minimum, *i.e.*, the minimum where the universe lives.

In this paper, we consider both  $Z_2$ -conserving and  $Z_2$ -violating 2HDM scalar potentials for our analysis. By  $Z_2$ -breaking 2HDM, we mean those with dimension-4 operators violating  $Z_2$ , like  $(\Phi_1^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2)$  or  $(\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2)$ . There can be soft  $Z_2$ -violating terms like  $\Phi_1^\dagger \Phi_2$  or  $\Phi_2^\dagger \Phi_1$  in the potential; if there are no dimension-4  $Z_2$ -breaking terms, we call those models  $Z_2$ -conserving, although, strictly speaking, they are not. We show that the conclusions drawn about the nature of the minima of the potential in the context of  $Z_2$ -conserving 2HDMs remain valid for  $Z_2$ -violating 2HDMs too.

To check the robustness of the tree-level results, we further perform a one-loop analysis of the 2HDM potential, and choose only those models that show a double minima. A nice review of the one-loop corrections in the context of  $Z_2$ -symmetric 2HDM and scale invariant 2HDM can be found in Refs. [26, 27, 28]. It is a common knowledge that the one-loop corrections [29] can be significant only in the flat direction of the potential. For the SM, this is easy to obtain [30]; so is for the Inert Doublet models <sup>3</sup> [32, 33] where one of the VEVs is zero. For a generic 2HDM, this is a cumbersome task, but can be done, in principle, following the prescriptions of Gildener and Weinberg [34], and the ray in the potential space along which the tree-level potential is zero can be found. However, If  $v_2 \gg v_1$ , where  $v_1$  and  $v_2$  are the VEVs of the  $CP$ -even neutral components of  $\Phi_1$  and  $\Phi_2$  respectively (so that  $\tan \beta \equiv v_2/v_1 \gg 1$ ), the potential along the  $\Phi_1$  direction is *almost* flat, so it is instructive to show the variations of the potential perpendicular to this direction, *i.e.*, along  $\Phi_2$ . Another important point is the setting of the regularization scale  $\mu$  for the one-loop corrections. Variation of  $\mu$  is equivalent to the variations of the tree-level quartic couplings  $\lambda_i$ , as can be seen from a renormalization group argument. As the nature of the potential can best be described by these quartic couplings, we would like to forward a prescription of choosing  $\mu$  for the 2HDM: choose it so that the position of the EW minimum remains unaltered. As we will see, this keeps the position of the second minimum too almost unaltered. Of course, the depths of the potential at the two minima will change. With such a prescription for choosing  $\mu$ , the conclusions about the stability of the EW minimum that were drawn from a tree-level analysis remain unaltered.

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<sup>2</sup> For example, the  $b \rightarrow s\gamma$  constraint,  $m_{H^\pm} > 316$  GeV, can be evaded in Type I and Lepton-specific 2HDM.

<sup>3</sup> For the Inert Doublet model (which is important from the cosmological implication of providing a cold dark matter candidate), one of the VEVs is zero, and so it is easier to treat analytically. We, however, will not go into any detailed study of such cosmological implications of the 2HDMs in this paper. Another such implication is the successful first-order electroweak phase transition and electroweak baryogenesis, which is discussed in Refs. [31].

The paper is arranged as follows. In section II we briefly review 2HDMs, with softly broken  $Z_2$  symmetry and without  $Z_2$  symmetry, and list the constraints and the minimization conditions on the potential. Section III introduces the one-loop corrected effective potential and modified minimization conditions for Type II 2HDM with and without  $Z_2$  symmetry. Our results, for both tree-level and one-loop analysis, are shown in Section IV. Section V summarizes the paper. Some calculation details and relevant expressions are relegated to the Appendix.

## 2 Brief review of 2HDM

### 2.1 2HDM with softly broken $Z_2$ symmetry

To start with, let us focus on the most canonical 2HDMs, where the  $Z_2$  symmetry is broken only softly by a dimension-2 operator, and all the couplings are real. The notations used here essentially follow those in Ref. [1]. Later on, we will introduce both dimension-4  $Z_2$ -breaking operators as well as complex parameters in the scalar potential.

Let us denote the two doublets, both with hypercharge  $Y = +1$ , by  $\Phi_1$  and  $\Phi_2$ , which can be written more explicitly as

$$\Phi_a = \begin{pmatrix} \chi_a^+ \\ \frac{1}{\sqrt{2}}(\phi_a + i\eta_a) \end{pmatrix}, \quad a = 1, 2. \quad (1)$$

We further assume the VEVs to be aligned towards the direction of the CP-even neutral field, so that  $\langle\phi_1\rangle = v_1$ ,  $\langle\phi_2\rangle = v_2$ , and we conventionally denote  $\tan\beta = v_2/v_1$ .

Invoking a  $Z_2$  symmetry  $\Phi_1 \rightarrow \Phi_1$ ,  $\Phi_2 \rightarrow -\Phi_2$ , so that there is no tree-level flavor-changing neutral current (FCNC), one may write

$$\begin{aligned} V(\Phi_1, \Phi_2) = & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) \\ & + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) \\ & + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \frac{1}{2} \lambda_5 \left[ (\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2 \right]. \end{aligned} \quad (2)$$

Here  $m_{11}^2$ ,  $m_{22}^2$ ,  $m_{12}^2$ ,  $\lambda_1$ ,  $\lambda_2$ ,  $\lambda_3$ ,  $\lambda_4$  and  $\lambda_5$  are all real and  $m_{12}^2$  softly breaks the  $Z_2$  symmetry. As mentioned before, to differentiate from the potential that breaks the  $Z_2$  symmetry with dimension-4 operators, such models will henceforth be called the  $Z_2$ -conserving or  $Z_2$ -symmetric models, even though it is broken softly by  $m_{12}^2$ .

The two CP-even neutral states  $\phi_1$  and  $\phi_2$  are in general not mass eigenstates. The corresponding mass matrix can be diagonalized through a rotation by an angle  $\alpha$ , and the mass eigenstates are

$$h = \phi_2 \cos\alpha - \phi_1 \sin\alpha, \quad H = \phi_2 \sin\alpha + \phi_1 \cos\alpha, \quad (3)$$

where  $h(H)$  is the lighter (heavier) eigenstate.

Note that if  $|\alpha - \beta|$  is an odd (even) multiple of  $\pi/2$ ,  $h(H)$  becomes identical with the SM Higgs boson, with a VEV of  $v = \sqrt{v_1^2 + v_2^2} \approx 246$  GeV. For example, the  $hVV^*$  ( $HVV^*$ ) coupling is just the SM coupling times  $\sin(\alpha - \beta)$  ( $\cos(\alpha - \beta)$ ), where  $V$  is any weak gauge boson,  $W$  or  $Z$ . The limit where  $h$  behaves as the SM Higgs boson is known as the alignment limit. The LHC data strongly favours the alignment limit and this sets a nontrivial constraint on the parameter space. The allowed parameter space, of course, depends on what type of 2HDM is chosen. We refer the reader to Ref. [14, 15] for a study of the alignment limit in Type I and Type II 2HDMs, and to Ref. [6] for a typical example of constraints coming from 2HDMs with tree-level FCNC.

The most generic Yukawa interactions for these four models can be written as [1],

$$\mathcal{L}_Y = - \sum_{j=1}^2 \left[ Y_j^d \bar{Q}_L d_R \Phi_j + Y_j^u \bar{Q}_L u_R \tilde{\Phi}_j + Y_j^e \bar{L}_L l_R \Phi_j + \text{h.c.} \right], \quad (4)$$

where  $\tilde{\Phi}_j = i\tau_2 \Phi_j^*$ ,  $Q_L$ ,  $L_L$ ,  $d_R$ ,  $u_R$  and  $l_R$  are generic doublet quarks, doublet leptons, singlet down-type and singlet up-type quarks, and singlet charged leptons respectively.  $Y_j^d$ ,  $Y_j^u$ ,  $Y_j^e$  are  $3 \times 3$  complex matrices, containing Yukawa couplings for the down, up, and leptonic sectors respectively. In our analysis we will consider only top, bottom, and  $\tau$  Yukawa couplings to be nonzero.

The masses of the charged Higgs,  $H^\pm$ , and the pseudoscalar,  $A$ , can be written as

$$\begin{aligned} m_{H^\pm}^2 &= \frac{1}{v_1 v_2} m_{12}^2 v^2 - \frac{1}{2} (\lambda_4 + \lambda_5) v^2, \\ m_A^2 &= \frac{1}{v_1 v_2} m_{12}^2 v^2 - \lambda_5 v^2. \end{aligned} \quad (5)$$

The couplings can be expressed in terms of masses of the physical states and the mixing angles  $\alpha$  and  $\beta$  as [20]

$$\begin{aligned} \lambda_1 &= \frac{1}{v^2 c_\beta^2} \left( c_\alpha^2 m_H^2 + s_\alpha^2 m_h^2 - \frac{m_{12}^2 s_\beta}{c_\beta} \right), \\ \lambda_2 &= \frac{1}{v^2 s_\beta^2} \left( s_\alpha^2 m_H^2 + c_\alpha^2 m_h^2 - \frac{m_{12}^2 c_\beta}{s_\beta} \right), \\ \lambda_3 &= \frac{2m_{H^\pm}^2}{v^2} + \frac{s_{2\alpha}}{v^2 s_{2\beta}} (m_H^2 - m_h^2) - \frac{m_{12}^2}{v^2 s_\beta c_\beta}, \\ \lambda_4 &= \frac{1}{v^2} (m_A^2 - 2m_{H^\pm}^2) + \frac{m_{12}^2}{v^2 s_\beta c_\beta}, \\ \lambda_5 &= \frac{m_{12}^2}{v^2 s_\beta c_\beta} - \frac{m_A^2}{v^2}. \end{aligned} \quad (6)$$

where  $m_h(m_H)$  is the mass of  $h(H)$ , and  $c_\theta$  and  $s_\theta$  are generic shorthand notations for  $\cos \theta$  and  $\sin \theta$  respectively.

The requirement that the scalar potential always remains bounded from below leads to the following stability conditions for 2HDMs with  $Z_2$  symmetry [1],

$$\lambda_1, \lambda_2 \geq 0, \quad \lambda_3 \geq -\sqrt{\lambda_1 \lambda_2}, \quad \lambda_3 + \lambda_4 - |\lambda_5| \geq -\sqrt{\lambda_1 \lambda_2}. \quad (7)$$

Note that  $\lambda_3$ ,  $\lambda_4$ , and  $\lambda_5$  can in principle be negative; however, if the  $Z_2$  symmetry is exact, this may lead to tachyonic masses for  $H^\pm$  and  $A$ .

## 2.2 2HDM with hard $Z_2$ breaking terms

The most general renormalizable scalar potential of 2HDM can be written as [1]

$$\begin{aligned} V(\Phi_1, \Phi_2) &= m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[ m_{12}^2 (\Phi_1^\dagger \Phi_2) + \text{h.c.} \right] \\ &\quad + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) \\ &\quad + \left[ \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \lambda_6 (\Phi_1^\dagger \Phi_1) (\Phi_1^\dagger \Phi_2) + \lambda_7 (\Phi_2^\dagger \Phi_2) (\Phi_1^\dagger \Phi_2) + \text{h.c.} \right], \end{aligned} \quad (8)$$

where the model parameters  $m_{11}^2$ ,  $m_{22}^2$ ,  $\lambda_1$ ,  $\lambda_2$ ,  $\lambda_3$ ,  $\lambda_4$  are real and  $m_{12}^2$ ,  $\lambda_5$ ,  $\lambda_6$  and  $\lambda_7$  can in principle be complex, and “h.c.” stands for hermitian conjugation.

We will follow the same convention as the previous subsection and consider two different cases, namely,  $m_{12}^2$ ,  $\lambda_5$ ,  $\lambda_6$  and  $\lambda_7$  are (i) real, and (ii) complex.

## 2.3 2HDMs without $Z_2$ symmetry with all real parameters

For this case, the masses of  $H^\pm$  and  $A$  can be written in an analogous way of Eq. (5), using the minimization conditions of the potential:

$$\begin{aligned} m_{H^\pm}^2 &= \frac{1}{v_1 v_2} m_{12}^2 v^2 - \frac{1}{2} (\lambda_4 + \lambda_5) v^2 - \frac{v_1}{2v_2} \lambda_6 v^2 - \frac{v_2}{2v_1} \lambda_7 v^2, \\ m_A^2 &= \frac{1}{v_1 v_2} m_{12}^2 v^2 - \lambda_5 v^2 - \frac{v_1}{2v_2} \lambda_6 v^2 - \frac{v_2}{2v_1} \lambda_7 v^2. \end{aligned} \quad (9)$$

The requirement that the scalar potential always remains bounded from below leads to the same set of equations as in (7), with an extra condition:

$$\begin{aligned} \lambda_1, \lambda_2 &\geq 0, \quad \lambda_3 \geq -\sqrt{\lambda_1 \lambda_2}, \quad \lambda_3 + \lambda_4 - |\lambda_5| \geq -\sqrt{\lambda_1 \lambda_2}, \\ 2|\lambda_6 + \lambda_7| &\leq \frac{1}{2} (\lambda_1 + \lambda_2) + \lambda_3 + \lambda_4 + \lambda_5. \end{aligned} \quad (10)$$

The mixing angle between the CP-even neutral states is given by  $\alpha = \frac{1}{2} \arctan(A/B)$ , where

$$\begin{aligned} A &= m_{12}^2 - (\lambda_3 + \lambda_4 + \lambda_5) v_1 v_2 - \frac{3}{2} \lambda_6 v_1^2 - \frac{3}{2} \lambda_7 v_2^2 \\ B &= -\frac{1}{2} \lambda_1 v_1^2 + \frac{1}{2} \lambda_2 v_2^2 + \frac{1}{2} m_{12}^2 (v_1/v_2 - v_2/v_1) + \frac{3}{4} (\lambda_7 - \lambda_6) v_1 v_2 + \frac{1}{4v_1} \lambda_7 v_2^3 - \frac{1}{4v_2} \lambda_6 v_1^3. \end{aligned} \quad (11)$$

### 2.3.1 Minima of the potential

The scalar potential of the 2HDM shows a much more complicated structure than that of the SM. The potential can have multiple non-equivalent normal stationary points (only two of them can be minima) [20, 22, 23, 24], depending on the parameters. These minima can be all normal (where only  $\phi_1$  and  $\phi_2$  get nonzero VEV), charge-breaking (where at least one of the charged fields  $\chi_a^+$  gets a nonzero VEV) or CP violating (where the two VEVs have a nontrivial phase between them). It has been shown in Refs. [20, 21, 22, 24], that for the canonical  $Z_2$ -symmetric 2HDM, (i) existence of a normal minimum rules out a charge breaking or CP violating minimum, those extrema can only be saddle points at best; (ii) there can be more than one normal minima, only one of which corresponds to the Standard model (SM), which we call the EW minimum. If the EW minimum is the global one, we are in a stable situation; if it is not, the universe may tunnel down to the deeper minimum if the tunneling time is less than or of the order of the lifetime of the universe. It has also been shown that the LHC data effectively rules out the parameter space where the EW minimum is shallower. We would like to extend this result to  $Z_2$ -breaking 2HDM with both real and complex parameters, and also investigate the nature of the potential when one-loop corrections are taken into account.

In a normal minimum, we may write

$$\langle \Phi_1 \rangle_N = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle_N = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}. \quad (12)$$

In certain situations, several non-equivalent normal extrema are allowed by the minimization conditions of the potentials and at most two of them can be local minima. Thus, there can be a second minimum with VEVs  $(v'_1, v'_2)$ , with  $v' = \sqrt{v'_1^2 + v'_2^2} \neq 246$  GeV, where exactly same symmetries are broken. Obviously, even with the same parameters of the potential, the masses of all SM particles are going to change from the EW vacuum configuration.

At the EW vacuum, the potential, as follows from Eq.(8), can be written as

$$\begin{aligned} V_0 &= \frac{1}{2} m_{11}^2 v_1^2 + \frac{1}{2} m_{22}^2 v_2^2 - m_{12}^2 v_1 v_2 + \frac{1}{8} \lambda_1 v_1^4 + \frac{1}{8} \lambda_2 v_2^4 + \frac{1}{4} (\lambda_3 + \lambda_4 + \lambda_5) v_1^2 v_2^2 \\ &\quad + \frac{1}{2} \lambda_6 v_1^3 v_2 + \frac{1}{2} \lambda_7 v_2^3 v_1, \end{aligned} \quad (13)$$

and at the other minimum, we get the corresponding  $V'_0$  by replacing  $(v_1, v_2)$  in Eq. (13) by  $(v'_1, v'_2)$ .

If  $V_0 < (>)V'_0$ , the EW (second) vacuum is stable and is the global minimum.

The minimization conditions of the potential are easy to obtain:

$$\begin{aligned} f_1(v_1, v_2) &\equiv m_{11}^2 v_1 - m_{12}^2 v_2 + \frac{1}{2} \lambda_1 v_1^3 + \frac{1}{2} (\lambda_3 + \lambda_4 + \lambda_5) v_1 v_2^2 + \frac{3}{2} \lambda_6 v_1^2 v_2 + \frac{1}{2} \lambda_7 v_2^3 = 0, \\ f_2(v_1, v_2) &\equiv m_{22}^2 v_2 - m_{12}^2 v_1 + \frac{1}{2} \lambda_2 v_2^3 + \frac{1}{2} (\lambda_3 + \lambda_4 + \lambda_5) v_2 v_1^2 + \frac{1}{2} \lambda_6 v_1^3 + \frac{3}{2} \lambda_7 v_2^2 v_1 = 0. \end{aligned} \quad (14)$$

with

$$\begin{aligned} \left[ m_{11}^2 + \frac{3}{2} \lambda_1 v_1^2 + \frac{1}{2} (\lambda_3 + \lambda_4 + \lambda_5) v_2^2 + 3 \lambda_6 v_1 v_2 \right] &> 0, \\ \left[ m_{22}^2 + \frac{3}{2} \lambda_2 v_2^2 + \frac{1}{2} (\lambda_3 + \lambda_4 + \lambda_5) v_1^2 + 3 \lambda_7 v_1 v_2 \right] &> 0. \end{aligned} \quad (15)$$

### 2.3.2 Charge-breaking minimum

If the symmetry breaks in such a way that one of the charged fields, say  $\chi_2^+$ , gets a non-zero VEV  $\gamma$  too, such that

$$\langle \Phi_1 \rangle_{CB} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle_{CB} = \frac{1}{\sqrt{2}} \begin{pmatrix} \gamma \\ v_2 \end{pmatrix}, \quad (16)$$

the minimization conditions of the potential are

$$\begin{aligned} m_{11}^2 v_1 - m_{12}^2 v_2 + \frac{1}{2} \lambda_1 v_1^3 + \frac{1}{2} \lambda_3 v_1 (v_2^2 + \gamma^2) + \frac{1}{2} (\lambda_4 + \lambda_5) v_1 v_2^2 + \frac{3}{2} \lambda_6 v_1^2 v_2 + \frac{1}{2} \lambda_7 (\gamma^2 + v_2^2) v_2 &= 0, \\ m_{22}^2 v_2 - m_{12}^2 v_1 + \frac{1}{2} \lambda_2 (v_2^2 + \gamma^2) v_2 + \frac{1}{2} (\lambda_3 + \lambda_4 + \lambda_5) v_2 v_1^2 + \frac{1}{2} \lambda_6 v_1^3 + \frac{1}{2} \lambda_7 (\gamma^2 + 3v_2^2) v_1 &= 0, \\ m_{22}^2 + \frac{1}{2} \lambda_2 (\gamma^2 + v_2^2) + \frac{1}{2} \lambda_3 v_1^2 + \lambda_7 v_1 v_2 &= 0, \end{aligned} \quad (17)$$

and

$$\begin{aligned} m_{11}^2 + \frac{3}{2} \lambda_1 v_1^2 + \frac{1}{2} \lambda_3 (\gamma^2 + v_2^2) + \frac{1}{2} (\lambda_4 + \lambda_5) v_2^2 + 3 \lambda_6 v_1 v_2 &> 0, \\ m_{22}^2 + \frac{1}{2} \lambda_2 (3v_2^2 + \gamma^2) + \frac{1}{2} (\lambda_3 + \lambda_4 + \lambda_5) v_1^2 + 3 \lambda_7 v_1 v_2 &> 0, \\ m_{22}^2 + \frac{1}{2} \lambda_2 (3\gamma^2 + v_2^2) + \frac{1}{4} \lambda_3 v_1^2 + \lambda_7 v_1 v_2 &> 0. \end{aligned} \quad (18)$$

### 2.3.3 CP violating minimum

At the CP violating extremum, one of the CP-odd neutral fields acquire a nonzero VEV  $\delta$ . In other words, the VEVs develop a relative phase. Thus,

$$\Phi_1 = \begin{pmatrix} \chi_1^+ \\ \frac{1}{\sqrt{2}} ((\phi_1 + v_1) + i(\eta_1 + \delta)) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \chi_2^+ \\ \frac{1}{\sqrt{2}} ((\phi_2 + v_2) + i\eta_2) \end{pmatrix}. \quad (19)$$

We have similar minimization conditions for the potential:

$$\begin{aligned} m_{11}^2 v_1 - m_{12}^2 v_2 + \frac{1}{2} \lambda_1 (v_1^2 + \delta^2) v_1 + \frac{1}{2} (\lambda_3 + \lambda_4 + \lambda_5) v_1 v_2^2 + \frac{1}{2} \lambda_6 (3v_1^2 + \delta^2) v_2 + \frac{1}{2} \lambda_7 v_2^3 &= 0, \\ m_{22}^2 v_2 - m_{12}^2 v_1 + \frac{1}{2} \lambda_2 v_2^3 + \frac{1}{2} (\lambda_3 + \lambda_4) v_2 (v_1^2 + \delta^2) + \frac{1}{2} \lambda_5 (v_1^2 - \delta^2) v_2 + \frac{1}{2} \lambda_6 (v_1^2 + \delta^2) v_1 \\ + \frac{3}{2} \lambda_7 v_1 v_2^2 &= 0, \\ m_{11}^2 \delta + \frac{1}{2} \lambda_1 (\delta^2 + v_1^2) \delta + \frac{1}{2} (\lambda_3 + \lambda_4) v_2^2 \delta - \frac{1}{2} \lambda_5 v_2^2 \delta + \lambda_6 v_1 v_2 \delta &= 0, \end{aligned} \quad (20)$$

and

$$\begin{aligned}
m_{11}^2 + \frac{1}{2}\lambda_1(3v_1^2 + \delta^2) + \frac{1}{2}(\lambda_3 + \lambda_4 + \lambda_5)v_2^2 + 3\lambda_6v_1v_2 &> 0, \\
m_{22}^2 + \frac{3}{2}\lambda_2v_2^2 + \frac{1}{2}(\lambda_3 + \lambda_4)(v_1^2 + \delta^2) + \frac{1}{2}\lambda_5(v_1^2 - \delta^2) + 3\lambda_7v_1v_2 &> 0, \\
m_{11}^2 + \frac{1}{2}\lambda_1(3v_1^2 + \delta^2) + \frac{1}{2}(\lambda_3 + \lambda_4 - \lambda_5)v_2^2 + \lambda_6v_1v_2 &> 0.
\end{aligned} \tag{21}$$

## 2.4 2HDM without $Z_2$ symmetry with complex parameters

Next we consider the 2HDMs with  $m_{12}^2$ ,  $\lambda_5$ ,  $\lambda_6$  and  $\lambda_7$  complex, and use

$$m_{12}^2 = m_{12}^{2R} + im_{12}^{2I}, \quad \lambda_5 = \lambda_{51} + i\lambda_{52}, \quad \lambda_6 = \lambda_{61} + i\lambda_{62}, \quad \lambda_7 = \lambda_{71} + i\lambda_{72}. \tag{22}$$

Most of the relevant expressions are identical with the real parameter case discussed before, with the  $(m_{12}^{2R}, \lambda_{51}, \lambda_{61}, \lambda_{71})$  replacing  $(m_{12}^2, \lambda_5, \lambda_6, \lambda_7)$  respectively. While this applies to the expressions of physical mass eigenstates, there is an extra condition so that the charged Goldstone boson is still massless:

$$-m_{12}^{2I} + \frac{1}{2}\lambda_{52}v_1v_2 + \frac{1}{2}\lambda_{62}v_1^2 + \frac{1}{2}\lambda_{72}v_2^2 = 0. \tag{23}$$

This shows that only three out of the four phases are independent.

The conditions for the stability of the potential are

$$\begin{aligned}
\lambda_1, \lambda_2 \geq 0, \quad \lambda_3 \geq -\sqrt{\lambda_1\lambda_2}, \quad \lambda_3 + \lambda_4 - \sqrt{\lambda_{51}^2 + \lambda_{52}^2} \geq -\sqrt{\lambda_1\lambda_2}, \\
2|\lambda_{61} + \lambda_{71}| \leq \frac{1}{2}(\lambda_1 + \lambda_2) + \lambda_3 + \lambda_4 + \lambda_{51}, \quad 2|\lambda_{62} + \lambda_{72}| \leq \frac{1}{2}(\lambda_1 + \lambda_2) + \lambda_3 + \lambda_4 - \lambda_{51}.
\end{aligned} \tag{24}$$

Similarly, the minimization conditions for the normal and charge-breaking minima are obtained from the corresponding expressions for the real parameter case by the substitutions mentioned before. For the charge-breaking minima, the imaginary parts of the potential parameters play a role. The minimization conditions are

$$\begin{aligned}
m_{11}^2v_1 - m_{12}^{2R}v_2 + \frac{1}{2}\lambda_1(v_1^2 + \delta^2)v_1 + \frac{1}{2}(\lambda_3 + \lambda_4 + \lambda_{51})v_1v_2^2 + \frac{1}{2}\lambda_{52}v_2^2\delta \\
+ \frac{1}{2}\lambda_{61}(3v_1^2 + \delta^2)v_2 + \lambda_{62}v_1v_2\delta + \frac{1}{2}\lambda_{71}v_2^3 &= 0, \\
m_{22}^2v_2 - m_{12}^{2R}v_1 - m_{12}^{2I}\delta + \frac{1}{2}\lambda_2v_2^3 + \frac{1}{2}(\lambda_3 + \lambda_4)v_2(v_1^2 + \delta^2) + \frac{1}{2}\lambda_{51}(v_1^2 - \delta^2)v_2 \\
+ \lambda_{52}v_1v_2\delta + \frac{1}{2}\lambda_{61}(v_1^2 + \delta^2)v_1 + \frac{1}{2}\lambda_{62}(v_1^2 + \delta^2)\delta + \frac{3}{2}\lambda_{71}v_1v_2^2 + \frac{3}{2}\lambda_{72}v_2^2\delta &= 0, \\
m_{11}^2\delta - m_{12}^{2I}v_2 + \frac{1}{2}\lambda_1(\delta^2 + v_1^2)\delta + \frac{1}{2}(\lambda_3 + \lambda_4)v_2^2\delta - \frac{1}{2}\lambda_{51}v_2^2\delta + \frac{1}{2}\lambda_{52}v_1v_2^2 \\
+ \lambda_{61}v_1v_2\delta + \frac{1}{2}\lambda_{62}(3\delta^2 + v_1^2)v_2 + \frac{1}{2}\lambda_{72}v_2^3 &= 0,
\end{aligned} \tag{25}$$

and

$$\begin{aligned}
m_{11}^2 + \frac{1}{2}\lambda_1(3v_1^2 + \delta^2) + \frac{1}{2}(\lambda_3 + \lambda_4 + \lambda_{51})v_2^2 + 3\lambda_{61}v_1v_2 + \lambda_{62}v_2\delta &> 0, \\
m_{22}^2 + \frac{3}{2}\lambda_2v_2^2 + \frac{1}{2}(\lambda_3 + \lambda_4)(v_1^2 + \delta^2) + \frac{1}{2}\lambda_{51}(v_1^2 - \delta^2) + \lambda_{52}v_1\delta + 3\lambda_{71}v_1v_2 + 3\lambda_{72}v_2\delta &> 0, \\
m_{11}^2 + \frac{1}{2}\lambda_1(3v_1^2 + \delta^2) + \frac{1}{2}(\lambda_3 + \lambda_4 - \lambda_{51})v_2^2 + \lambda_{61}v_1v_2 + 3\lambda_{62}v_2\delta &> 0.
\end{aligned} \tag{26}$$

The tree-level analysis will be performed using these expressions and the corresponding solutions of the simultaneous minimization equations.

### 3 One-loop effective potential

The form of the one-loop correction to the scalar potential is quite standard, and is given by

$$m_i \rightarrow m_i(\phi_{c1}, \phi_{c2}), \quad V_1 = \frac{1}{64\pi^2} \sum_{i=B,F} N_i m_i^4 \left( \ln \frac{m_i^2}{\mu^2} - C_i \right), \quad (27)$$

where the sum runs over all bosonic (the physical scalars, the unphysical Goldstone bosons,  $W$  and  $Z$ ) and fermionic (for our case, only  $t$ ,  $b$  and  $\tau$ ) degrees of freedom. Here  $\mu$  is the regularization scale. The masses are field-dependent quantities, being functions of the background fields  $\phi_{c1}$  and  $\phi_{c2}$ . They can be thought as the positions of the minima in the field space. In the limit  $\phi_{c1} = v_1$ ,  $\phi_{c2} = v_2$ , the field-dependent masses are equal to the physical masses. The constants  $N_i$  and  $C_i$  are given by

$$\begin{aligned} N_h = N_H = N_A = N_{G^0} &= 1, \quad N_{H^\pm} = N_{G^\pm} = 2, \\ N_W = 6, \quad N_Z = 3, \quad N_t = N_b &= -12, \quad N_\tau = -4, \\ C_h = C_H = C_A = C_{H^\pm} = C_{G^0} = C_{G^\pm} &= C_t = C_b = C_\tau = \frac{3}{2}, \quad C_W = C_Z = \frac{5}{6}. \end{aligned} \quad (28)$$

The full potential can be written as

$$V = V(\Phi_1, \Phi_2) + V_1 \quad (29)$$

where  $V(\Phi_1, \Phi_2)$  is the tree-level potential as given in Eq. (2) or (8).

At this point, let us again note that the masses in Eq. (27) are functions of background fields  $\phi_{c1}$  and  $\phi_{c2}$ . The positions of the minima are functions of not only  $\phi_{c1}$  and  $\phi_{c2}$  but also the regularization scale  $\mu$ . To have any idea of the nature of the potential after one-loop correction, one has to fix  $\mu$  by some prescription. The renormalization group improved one-loop potential clearly shows that the variation in  $\mu$  is equivalent to the redefinition of the coupling parameters of the theory. One popular way is to fix it in such a way that the one-loop corrections are minimum, hoping that this will minimize the higher-order corrections too. To be physically more transparent, we try a different approach: we fix  $\mu$  in such a way that the position of the EW minimum remains (almost) unchanged with respect to the tree-level position. This will keep the one-loop corrected field-dependent masses to be at the same values of the tree-level masses at the EW vacuum. Only the depth of the potential changes by the one-loop correction. Thus, our prescription is to tune  $\mu$  in such a way that  $\phi_{c1} = v_1$  and  $\phi_{c2} = v_2$  (so that, for example, the Goldstone bosons are still massless<sup>4</sup>). As we will show later, if one tunes  $\mu$  so that the position of the EW minimum is unchanged, it will keep the position of the second minimum almost unchanged too.

As a concrete example, let us now discuss the Type II 2HDM; as the Yukawa couplings enter the picture, one needs to specify the type of 2HDM under consideration. We have checked that the qualitative features remain unchanged in all other 2HDMs.

#### 3.1 Type-II 2HDM without $Z_2$ : One-loop corrected potential

To keep the discussion as much general as possible, let us focus on the one-loop correction to the 2HDM (without  $Z_2$  symmetry) tree level potential. We can obtain the same for  $Z_2$  symmetric 2HDM by putting  $\lambda_6$  and  $\lambda_7$  to be equal to zero, and by making  $\lambda_5$  and  $m_{12}^2$  real. Using the definitions of  $f_i(v_1, v_2)$  from Eq. (14), we can write the modified minimization condition for one-loop corrected potential  $V$  as

$$f_1(v_1, v_2) + \frac{\partial V_1}{\partial \phi_1} \bigg|_{\phi_{c1}=v_1} = 0, \quad f_2(v_1, v_2) + \frac{\partial V_1}{\partial \phi_2} \bigg|_{\phi_{c2}=v_2} = 0, \quad (30)$$

<sup>4</sup>The treatment of the Goldstone bosons in one-loop corrected potentials is tricky, and a consistent treatment needs resummation of the Goldstone contributions in the effective potential [35].

where,

$$\begin{aligned}
\left. \frac{\partial V_1}{\partial \phi_1} \right|_{\phi_{c1}=v_1} = & \frac{1}{64\pi^2} \left[ 4m_h^2 F(h, 1) \left( \ln \frac{m_h^2}{\mu^2} - 1 \right) + 4m_H^2 F(H, 1) \left( \ln \frac{m_H^2}{\mu^2} - 1 \right) \right. \\
& + 4m_A^2 \left( 1 - \ln \frac{m_A^2}{\mu^2} \right) \left( v_1 \lambda_{51} - \frac{m_{12}^{2R}}{2v_2} \left( 1 - \frac{v_2^2}{v_1^2} \right) + \frac{\lambda_{61} v^2}{4v_2} + \frac{\lambda_{61} v_1^2}{2v_2} - \frac{\lambda_{71} v_2 v^2}{4v_1^2} + \frac{\lambda_{71} v_2}{2} \right) \\
& + 4m_{H^\pm}^2 \left( 1 - \ln \frac{m_{H^\pm}^2}{\mu^2} \right) \left( v_1 \lambda_{45} - \frac{m_{12}^{2R}}{v_2} \left( 1 - \frac{v_2^2}{v_1^2} \right) + \frac{\lambda_{61} v^2}{2v_2} + \frac{\lambda_{61} v_1^2}{v_2} - \frac{\lambda_{71} v_2 v^2}{2v_1^2} + \lambda_{71} v_2 \right) \\
& - 6g_2^2 m_W^2 v_1 \left( \frac{1}{3} - \ln \frac{m_W^2}{\mu^2} \right) - 3g^2 m_Z^2 v_1 \left( \frac{1}{3} - \ln \frac{m_Z^2}{\mu^2} \right) + 24Y_b^2 m_b^2 v_1 \left( 1 - \ln \frac{m_b^2}{\mu^2} \right) \\
& \left. + 8Y_\tau^2 m_\tau^2 v_1 \left( 1 - \ln \frac{m_\tau^2}{\mu^2} \right) \right] \quad (31)
\end{aligned}$$

and

$$\begin{aligned}
\left. \frac{\partial V_1}{\partial \phi_2} \right|_{\phi_{c2}=v_2} = & \frac{1}{64\pi^2} \left[ 4m_h^2 F(h, 2) \left( \ln \frac{m_h^2}{\mu^2} - 1 \right) + 4m_H^2 F(H, 2) \left( \ln \frac{m_H^2}{\mu^2} - 1 \right) \right. \\
& + 4m_A^2 \left( 1 - \ln \frac{m_A^2}{\mu^2} \right) \left( v_2 \lambda_{51} - \frac{m_{12}^{2R}}{2v_1} \left( 1 - \frac{v_1^2}{v_2^2} \right) + \frac{\lambda_{71} v^2}{4v_1} + \frac{\lambda_{71} v_2^2}{2v_1} - \frac{\lambda_{61} v_1 v^2}{4v_2^2} + \frac{\lambda_{61} v_1}{2} \right) \\
& + 4m_{H^\pm}^2 \left( 1 - \ln \frac{m_{H^\pm}^2}{\mu^2} \right) \left( v_2 \lambda_{45} - \frac{m_{12}^{2R}}{v_1} \left( 1 - \frac{v_1^2}{v_2^2} \right) + \frac{\lambda_{71} v^2}{2v_1} + \frac{\lambda_{71} v_2^2}{v_1} - \frac{\lambda_{61} v_1 v^2}{2v_2^2} + \lambda_{61} v_1 \right) \\
& - 6g_2^2 m_W^2 v_2 \left( \frac{1}{3} - \ln \frac{m_W^2}{\mu^2} \right) - 3g^2 m_Z^2 v_2 \left( \frac{1}{3} - \ln \frac{m_Z^2}{\mu^2} \right) + 24Y_t^2 m_t^2 v_2 \left( 1 - \ln \frac{m_t^2}{\mu^2} \right) \right], \quad (32)
\end{aligned}$$

with  $\lambda_{45} = \lambda_4 + \lambda_{51}$  and  $g = \sqrt{g_1^2 + g_2^2}$ ,  $g_1$  and  $g_2$  being the  $U(1)_Y$  and  $SU(2)_L$  gauge couplings. Yukawa couplings for  $t$ ,  $b$ , and  $\tau$  are denoted by  $Y_t$ ,  $Y_b$  and  $Y_\tau$  respectively. The  $F$ -functions have been defined in Appendix A. Note that these expressions are valid only if we tune  $\mu$  to keep the position of the EW minimum of the one-loop corrected effective potential  $V$ , defined in Eq. (29), unchanged with respect to its position in the tree level potential. One also needs to check the second derivatives to ensure that the extremum is a local minimum:

$$\left. \frac{\partial^2 V}{\partial \phi_1^2} \right|_{\phi_{c1}=v_1} > 0, \quad \left. \frac{\partial^2 V}{\partial \phi_2^2} \right|_{\phi_{c2}=v_2} > 0. \quad (33)$$

The expressions for second derivatives are also given in Appendix A. Note the absence of the Goldstone bosons in Eqs. (30,31,32) because of our choice of  $\mu$  which keeps them massless.

## 4 Analysis and Results

### 4.1 2HDM at tree-level

From a random scan over the parameter space spanning over  $7 \times 10^8$  different choices of model parameters, we generate a number of models for which the following conditions are satisfied. Our analysis includes the canonical 2HDM with  $Z_2$  symmetry, and 2HDM without  $Z_2$  with both real and complex parameters.

- The potential has to be stable at all scales before it either blows up (due to one or more couplings hitting the Landau pole) or becomes unbounded from below.
- The dimensionless couplings must remain perturbative over the entire range of validity of the theory, except maybe at the very end where they approach the Landau pole.

- There should be one minimum of the scalar potential for which  $v = \sqrt{v_1^2 + v_2^2} = 246$  GeV. This we will call the EW vacuum. This, in fact, acts as the tightest constraint and rules out the largest chunk of randomly generated models. We focus only on those models that allow another local minimum apart from the EW vacuum. Both tree-level minimization conditions, for  $v_1$  and  $v_2$ , should be satisfied in both the vacua.
- Other constraints like that on the charged Higgs boson mass coming from  $b \rightarrow s\gamma$  are satisfied. We use  $m_{H^\pm} > 316$  GeV. While not all constraints are valid for all 2HDMs, we focus only on the Type II 2HDM.
- The 125 GeV resonance found at the Large Hadron Collider must have properties close to that of the SM Higgs boson. In other words, the alignment limit should be maintained. We have kept  $|\alpha - \beta|$  to be between  $0.9\pi/2$  and  $1.1\pi/2$ , noting that this is a rather conservative limit.

The ranges for our scan is as follows:

$$0.0 \leq \lambda_1, \lambda_2 \leq 1.0, \quad -1.0 \leq \lambda_3, \lambda_4, \lambda_5 \leq 1.0, \quad m_{12}^2 \leq 4 \times 10^6 \text{ GeV}^2, \quad 1.0 \leq \tan \beta \leq 50.0, \quad (34)$$

where the parameters are taken to be at the EW scale.

Apart from this, for the  $Z_2$ -violating cases, we have taken

$$-1.0 \leq \lambda_6, \lambda_7 \leq 1.0 \text{ (real couplings)}, \quad -1.0 \leq \lambda_{51}, \lambda_{52}, \lambda_{61}, \lambda_{62}, \lambda_{71}, \lambda_{72} \leq 1.0 \text{ (complex couplings)}. \quad (35)$$

Also, for this case the scan is made on  $m_{12}^2 = m_{12}^{2R} + im_{12}^{2I}$ , as the phase in  $m_{12}^2$  is fixed by Eq. (23).

We find a few common characteristics for these models that allow two minima. They are:

- (i) The EW vacuum is always deeper than the other vacuum. This happens mostly because of the imposition of the experimental constraints. Thus, even with the introduction of  $Z_2$ -breaking parameters, there is no chance of tunneling to the other minimum, at least at the tree-level. This reinforces the conclusions obtained by the authors of Ref. [20].
- (ii) If both the vacua are normal, there is no other minimum that breaks charge conservation or  $CP$ . This, again, is in tune of what the authors of Refs. [22, 24] found.

## 4.2 2HDM at one-loop level

We would now like to perform the same analysis on the one-loop corrected potential on those models that satisfy the initial constraints and show the presence of a double minima at the tree-level. The regularization scale  $\mu$  is so chosen as to make  $\phi_{c1} = v_1$  and  $\phi_{c2} = v_2$ . This is, of course, a rather restrictive choice, but keeps all the masses as well as  $\tan \beta$  unchanged at the EW minimum even after the one-loop corrections are implemented.

In a generic 2HDM where both CP-even neutral fields can get nonzero VEV, this is a tricky job, and mathematically much more complicated than the cases of only SM, or SM extended by a gauge singlet scalar, or the inert doublet models where one VEV is always zero. The complication is further enhanced by the fact that at the tree-level, there are two minima of the potential.

What we do is the following. We fix  $\phi_{c1} = v_1$  and tune  $\mu$  in such a way that  $\phi_{c2}$  coincides with  $v_2$ . This keeps the position of the EW minimum invariant but changes the depth. We could have done this the other way round too, namely, keeping  $\phi_{c2} = v_2$  and adjusting  $\mu$  to make  $\phi_{c1}$  coincide with the tree level value; however, we prefer the first approach as the contribution of  $v_2$  is larger in  $v$  for  $\tan \beta > 1$ . We then use the same  $\mu$  but fix the classical minimum for  $\phi_1$  at <sup>5</sup>  $\phi'_{c1} = v'_1$ . It so happens that  $\phi'_{c2}$ , in all cases, lies close to  $v'_2$ ; the coincidence is not as exact as the EW vacuum, but this being the shallower secondary minimum, we will not be so much bothered about its exact position. Thus, we

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<sup>5</sup>We use primes to denote the corresponding quantities in the second minimum.

compare  $V(\mu, \phi_{c1} = v_1, \phi_{c2} = v_2)$  with  $V'(\mu, \phi'_{c1} = v'_1, \phi'_{c2} \approx v'_2)$ . We will also show the effect of varying the scale  $\mu$  on the classical field values.

What we observe is that the conclusions drawn from a tree-level analysis is more or less unchanged; the deeper minimum remains deeper. This is not entirely unexpected, as the one-loop corrections are only a small effect that cannot overcome the difference in depths of these two minima. For  $\tan \beta \gg 1$ ,  $\phi_1$  direction is almost flat, so we show our results in the constant- $\phi_1$  direction, varying the potential  $V$  with  $\phi_2$ .

Our results are discussed in more detail later for some typical benchmark points, but for both the cases that we consider ( $Z_2$  conserving and breaking), we never found the second minimum becoming deeper than the EW one after the one-loop corrections. Thus, if the EW vacuum is the deeper one at tree-level, it remains so after the radiative corrections; there is no chance of developing a deeper vacuum and tunnelling into it. In fact, if  $\mu$  is tuned in such a way that the EW vacuum is not shifted from its tree-level position, *it always gets deeper by the one-loop corrections*;  $V_1$  at the EW vacuum is always negative.

### 4.3 One-loop corrected 2HDM with $Z_2$ symmetry

In Table 1, we show three benchmark points for the  $Z_2$ -conserving Type-II 2HDM, characterized by small, medium, and large values of  $\tan \beta (= v_2/v_1)$  respectively. All these models show the existence of a second and shallower minimum compared to the EW one. These three models, namely,  $Z_2C1$ ,  $Z_2C2$ , and  $Z_2C3$  are valid up to  $3.8 \times 10^7$  GeV,  $2.8 \times 10^{11}$  GeV, and the Planck scale respectively; for the first two models, at least one of the couplings become nonperturbative at the validity scale and the model soon hits the Landau pole thereafter. Obviously, the comparatively low validity range for the first benchmark can be ascribed to the relatively large quadratic couplings to start with at the EW scale. The evolutions are checked with one-loop renormalization group equations for the Type II 2HDM [1, 36].

Parameter	Benchmark		
	$Z_2C1$	$Z_2C2$	$Z_2C3$
$\lambda_1$	0.413	0.642	0.068
$\lambda_2$	0.842	0.328	0.260
$\lambda_3$	-0.265	0.065	0.132
$\lambda_4$	-0.720	-0.365	-0.489
$\lambda_5$	-0.929	-0.786	-0.498
$m_{11}^2$ (GeV $^2$ )	$2.3 \times 10^6$	$6.2 \times 10^5$	$2.61 \times 10^5$
$m_{22}^2$ (GeV $^2$ )	$2.8 \times 10^5$	$5.2 \times 10^3$	$-7.77 \times 10^3$
$m_{12}^2$ (GeV $^2$ )	$8.10 \times 10^5$	$9.11 \times 10^4$	$4.84 \times 10^3$
$v_1$ (GeV)	83.76	37.65	5.05
$v_2$ (GeV)	231.30	243.11	245.95
$v'_1$ (GeV)	442.46	452.48	399.18
$v'_2$ (GeV)	872.82	953.66	775.94

Table 1: Benchmark points for  $Z_2$ -conserving Type-II 2HDM.

The positions of the one-loop corrected minima and the corresponding regularization scales are shown in Table 2. The benchmarks are chosen scanning the range of  $\tan \beta$  as well as the other parameters.

The potential profiles are shown in Figure 1. These are drawn as a section of the actual three-

	$\tan \beta$	$\phi_{c1}$	$\phi_{c2}$	$\mu$	$\phi'_{c1}$	$\phi'_{c2}$
$Z_2C1$	2.76	83.76	231.30	976.0	442.46	846.7
$Z_2C2$	6.46	37.65	243.11	492.5	452.48	932.6
$Z_2C3$	48.7	5.05	245.95	335.0	953.66	774.6

Table 2: The one-loop corrected minima (all quantities except  $\tan\beta$  are in GeV) for the three benchmarks. Note the tiny shift of  $\phi'_{c2}$  from  $v'_2$ .

dimensional plots, for fixed values of  $\phi_{c1}$ . That is why the second minimum is not apparent; it occurs at a different value of  $\phi_{c1}$ . Also, the tree-level potential does not go to zero as  $\phi_{c2} \rightarrow 0$ , unless  $\phi_{c1}$  is tiny, as in  $Z_2C3$ . Note that the one-loop corrected potential is always deeper than the tree-level potential at the EW minimum; Table 2 shows that the second minimum  $\phi'_{c2}$  almost coincides with  $v'_2$ .

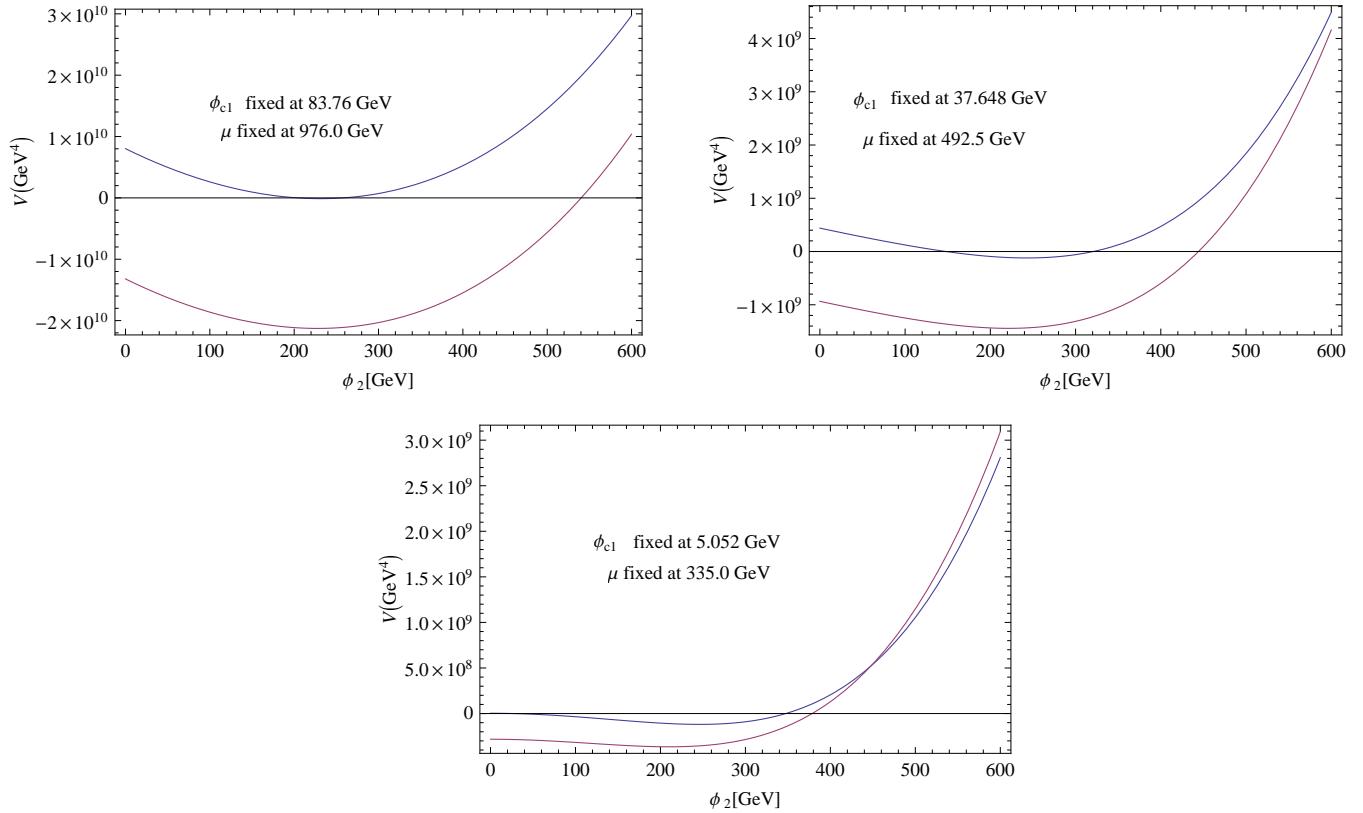


Figure 1: The plots of the tree-level and one-loop corrected potential, with the section taken at a fixed value of  $\phi_{c1}$  as indicated on the plots and in Table 2. The upper panel plots are for benchmarks  $Z_2C1$  (L) and  $Z_2C2$  (R), while the lower panel plot is for  $Z_2C3$ . In every plot, the upper curve (blue) denotes the tree-level potential profile, while the lower one (red) is for the one-loop corrected potential.

Only if  $\phi_{c1}$  is small, like in  $Z_2C3$ , the  $\phi_1$  direction can be approximated by a flat direction. The flatness is really impressive: for a 10% (1%) change in  $\phi_{c1}$ , the potential changes only by 0.02% ( $2.5 \times 10^{-4}\%$ ). However, we have not found any case where the one-loop corrections remove the second minimum.

If we keep  $\phi_{c1} = v_1$  or  $v'_1$ , and play with  $\mu$  as a free parameter,  $\phi_{c2}$  and  $\phi'_{c2}$  changes. In Fig. 2, we show how  $\phi_{c2}$  and  $\phi'_{c2}$  change with  $\mu$  for the three benchmarks.

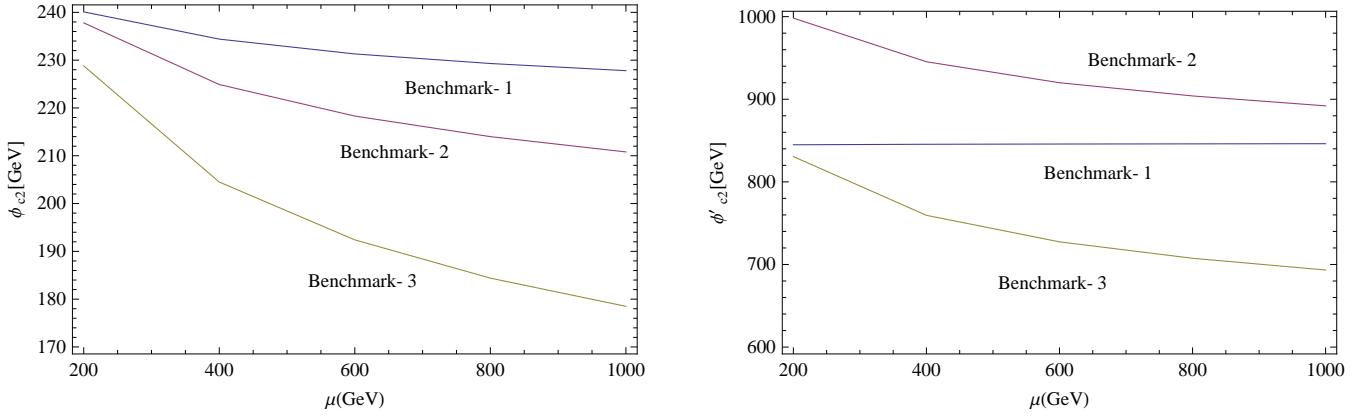


Figure 2: Variation of  $\phi_{c2}$  (L) and  $\phi'_{c2}$  (R) with  $\mu$  for  $Z_2C1$  (blue),  $Z_2C2$  (red), and  $Z_2C3$  (golden).

#### 4.4 One-loop corrected 2HDM without $Z_2$ symmetry

The analysis is analogous to what was performed for the  $Z_2$ -symmetric case. The potential profiles are shown in Fig. 3. For the three benchmark points  $Z_2V1$ ,  $Z_2V2$ , and  $Z_2V3$ , the  $\mu$ -values are fixed at 641.0 GeV, 655.3 GeV, and 2228 GeV respectively. While  $\phi_{c1} = v_1$ ,  $\phi_{c2} = v_2$ , and  $\phi'_{c1} = v'_1$  were ensured, the  $\phi'_{c2}$  values were quite close to that of  $v'_2$ ; they are at 506.2 GeV, 708.9 GeV, and 1783 GeV respectively.

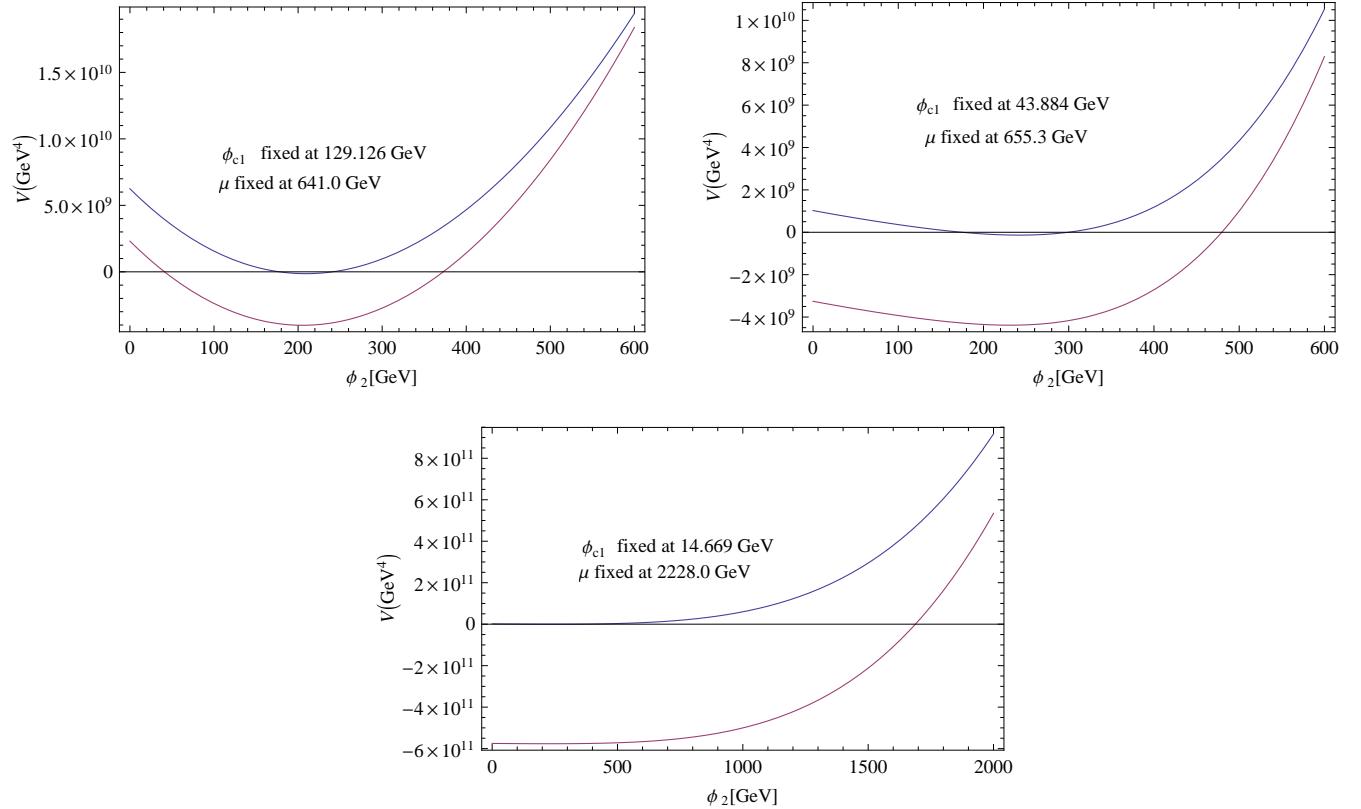


Figure 3: The plots of the tree-level and one-loop corrected potential, with the section taken at a fixed value of  $\phi_{c1}$  as indicated on the plots. The upper panel plots are for benchmarks  $Z_2V1$  (L) and  $Z_2V2$  (R), while the lower panel plot is for  $Z_2V3$ . In every plot, the upper curve (blue) denotes the tree-level potential, while the lower one (red) is for the one-loop corrected potential.

Parameter	Benchmark		
	$Z_2V1$	$Z_2V2$	$Z_2V3$
$\lambda_1$	0.656	0.342	0.497
$\lambda_2$	0.188	0.928	0.456
$\lambda_3$	0.836	0.998	0.089
$\lambda_4$	0.659	0.375	0.598
$\lambda_{51}$	0.764	-0.956	-0.533
$\lambda_{52}$	0.163	0.666	0.923
$\lambda_{61}$	0.633	0.735	0.680
$\lambda_{62}$	0.209	0.619	-0.929
$\lambda_{71}$	-0.820	-0.911	-0.810
$\lambda_{72}$	-0.0016	0.709	0.774
$m_{11}^2$ (GeV $^2$ )	$7.47 \times 10^5$	$1.06 \times 10^6$	$1.34 \times 10^7$
$m_{22}^2$ (GeV $^2$ )	$3.1 \times 10^5$	$1.78 \times 10^4$	$3.71 \times 10^4$
$m_{12}^{2R}$ (GeV $^2$ )	$4.92 \times 10^5$	$1.71 \times 10^5$	$7.78 \times 10^5$
$m_{12}^{2I}$ (GeV $^2$ )	$3.92 \times 10^3$	$2.49 \times 10^4$	$2.49 \times 10^4$
$v_1$ (GeV)	129.13	43.89	14.67
$v_2$ (GeV)	209.39	242.05	245.56
$v'_1$ (GeV)	295.20	257.06	355.55
$v'_2$ (GeV)	631.54	799.49	2064.3

Table 3: Benchmark points for  $Z_2$ -violating Type-II 2HDM. Note that  $\lambda_i = \lambda_{i1} + i\lambda_{i2}$  for  $i = 5, 6, 7$ . Also,  $m_{12}^2 = m_{12}^{2R} + im_{12}^{2I}$ .

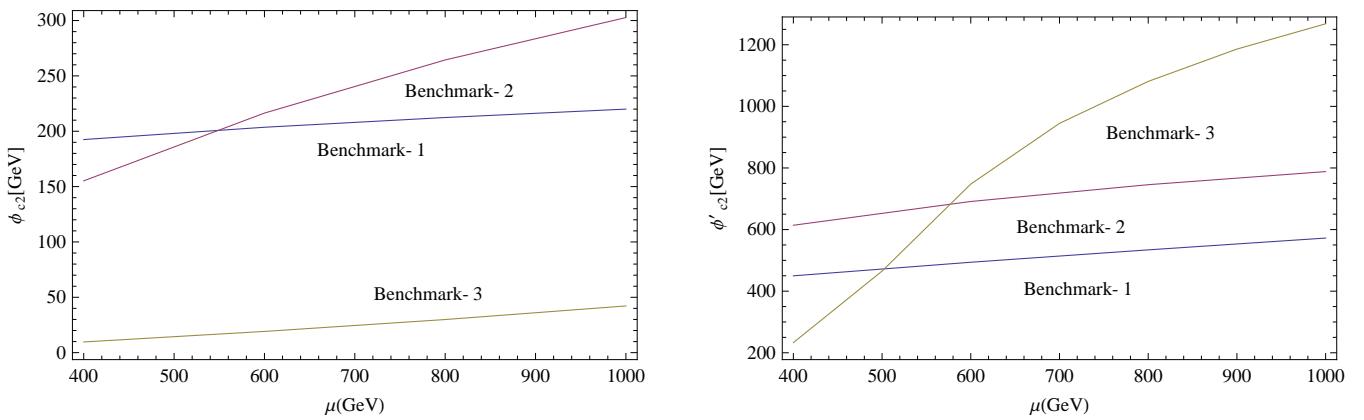


Figure 4: Variation of  $\phi_{c2}$  (L) and  $\phi'_{c2}$  (R) with  $\mu$  for  $Z_2V1$  (blue),  $Z_2V2$  (red), and  $Z_2V3$  (golden).

Just like the  $Z_2$ -conserving case, we choose three benchmarks for the case where the potential violates the  $Z_2$  and also involves complex parameters. These benchmarks are shown in Table 3.

The flatness of the potential in the  $\phi_1$  direction is again most manifest for  $Z_2V3$ , the benchmark with lowest  $v_1$ . At the same time, the EW minimum always gets deeper with the one-loop corrected potential, and there is no qualitative change from the tree-level result. Because of the comparatively large values of the couplings, all the three models tend to hit the Landau pole much before the Planck scale, namely, at 45.8 TeV, 4.8 TeV, and 216.7 TeV. This is expected because  $Z_2V2$  starts with larger values of the quartic couplings at the EW scale. In general, for the double-minima case when  $Z_2$  is broken, most of the couplings at the EW scale have to be large to start with and thus such class of models are not stable beyond a few hundreds of TeV, which may be contrasted with the  $Z_2$ -conserving double-minima models.

In Fig. 4, we show how  $\phi_{c2}$  and  $\phi'_{c2}$  changes with  $\mu$  for the three benchmarks.

## 5 Summary

In this paper we have tried to investigate the nature of the potential of Type II 2HDM, breaking the  $Z_2$  symmetry either softly or through dimension-4 operators. There are some known results for the  $Z_2$ -symmetric 2HDM at the tree level. Our goal was to investigate how far these conclusions are reliable if one (i) breaks the  $Z_2$  symmetry at the tree level with operators with real or complex couplings, (ii) does a one-loop correction on the potential.

For the first part, we find that the introduction of  $Z_2$  breaking does not change the conclusions qualitatively: the scalar potential can accommodate at most two local minima, both of which have to be normal. If there is a normal minimum, there cannot be a charge-breaking or  $CP$  violating minimum of the potential. The LHC data highly disfavours those models where the second minimum is deeper than the EW minimum, possibly making the EW vacuum an unstable or metastable one.

For one-loop corrections, we use a regularization scale that keeps the position of the EW minimum invariant, changing only its depth. When we focus on models with two minima, this prescription keeps the position of the second minima almost unchanged too. The one-loop corrections cannot change the relative depths of the minima, *i.e.*, the EW minimum still remains deeper after the correction, ruling out the possibility of a metastable vacuum. The conclusions are identical for  $Z_2$  symmetric and  $Z_2$  breaking 2HDM. While the conclusions were drawn for Type II 2HDM, the results are qualitatively the same for other 2HDMs too, as the only change comes from the Yukawa couplings that enter the one-loop corrections.

The stability of the 2HDM at higher energy scales is a complex issue because of more fields and couplings. One needs the stability conditions to be valid at all scales and the couplings to remain perturbative for calculability (or, at least, not blow up). A general tendency that can intuitively be deduced from the RG equations is that higher values of quartic couplings at the EW scale tend to pull down the range of validity of the theory, which means that some other ultraviolet complete theory takes hold beyond that range. However, a large part of the parameter space is still compatible with the stability up to the Planck scale for the  $Z_2$ -conserving class of models. For the  $Z_2$ -violating class, the existence of two minima generally forces some of the quartic couplings to be large at the EW scale and hence such models cease to be valid beyond a few hundreds of TeV at the most; for smaller couplings, one gets the single-minimum models.

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## A Expressions for first and second derivatives

We use the following shorthand notations:

$$F(\alpha, i) = m_\alpha \left( \frac{\partial m_\alpha}{\partial \phi_i} \right) \Big|_{\phi_{ci}=v_i}, \quad G(\alpha, i) = \left[ \left( \frac{\partial m_\alpha}{\partial \phi_i} \right)^2 + m_\alpha \left( \frac{\partial^2 m_\alpha}{\partial \phi_i^2} \right) \right]_{\phi_{ci}=v_i}. \quad (\text{A.1})$$

$$\begin{aligned}
F(h, 1) &= \frac{1}{4} \left( 2\lambda_1 v_1 + \frac{m_{12}^{2R}}{v_1^2 v_2} (v_1^2 - v_2^2) + \frac{3\lambda_{67} v_2}{2} + \frac{\lambda_{71} v_2^3}{2v_1^2} - \frac{3\lambda_{61} v_1^2}{2v_2} \right) \\
&\quad - \frac{1}{4\sqrt{a}} \left( \lambda_1 v_1^2 - \lambda_2 v_2^2 - \frac{m_{12}^{2R}}{v_1 v_2} (v_1^2 - v_2^2) + \frac{3v_1 v_2 (\lambda_{61} - \lambda_{71})}{2} - \frac{\lambda_{71} v_2^3}{2v_1} + \frac{\lambda_{61} v_1^3}{2v_2} \right) \times \\
&\quad \left( \lambda_1 v_1 - \frac{m_{12}^{2R}}{v_2} + (v_1^2 - v_2^2) \frac{m_{12}^{2R}}{2v_1^2 v_2} + \frac{3v_2 (\lambda_{61} - \lambda_{71})}{4} + \frac{\lambda_{71} v_2^3}{4v_1^2} + \frac{3\lambda_{61} v_1^2}{4v_2} \right) \\
&\quad - \frac{1}{2\sqrt{a}} \left( -m_{12}^{2R} + \lambda_{345} v_1 v_2 + \frac{3\lambda_{61} v_1^2}{2} + \frac{3\lambda_{71} v_2^2}{2} \right) \times (\lambda_{345} v_2 + 3\lambda_{61} v_1), \\
F(h, 2) &= \frac{1}{4} \left( 2\lambda_2 v_2 + \frac{m_{12}^{2R}}{v_2^2 v_1} (v_2^2 - v_1^2) + \frac{3\lambda_{67} v_1}{2} - \frac{3\lambda_{71} v_2^2}{2v_1} + \frac{\lambda_{61} v_1^3}{2v_2^2} \right) \\
&\quad - \frac{1}{4\sqrt{a}} \left( \lambda_1 v_1^2 - \lambda_2 v_2^2 - \frac{m_{12}^{2R}}{v_1 v_2} (v_1^2 - v_2^2) + \frac{3v_1 v_2 (\lambda_{61} - \lambda_{71})}{2} - \frac{\lambda_{71} v_2^3}{2v_1} + \frac{\lambda_{61} v_1^3}{2v_2} \right) \times \\
&\quad \left( -\lambda_2 v_2 + \frac{m_{12}^{2R}}{v_1} + (v_1^2 - v_2^2) \frac{m_{12}^{2R}}{2v_2^2 v_1} + \frac{3v_1 (\lambda_{61} - \lambda_{71})}{4} - \frac{3\lambda_{71} v_2^2}{4v_1} - \frac{\lambda_{61} v_1^3}{4v_2^2} \right) \\
&\quad - \frac{1}{2\sqrt{a}} \left( -m_{12}^{2R} + \lambda_{345} v_1 v_2 + \frac{3\lambda_{61} v_1^2}{2} + \frac{3\lambda_{71} v_2^2}{2} \right) \times (\lambda_{345} v_1 + 3\lambda_{71} v_2), \\
F(H, 1) &= \frac{1}{4} \left( 2\lambda_1 v_1 + \frac{m_{12}^{2R}}{v_1^2 v_2} (v_1^2 - v_2^2) + \frac{3\lambda_{67} v_2}{2} + \frac{\lambda_{71} v_2^3}{2v_1^2} - \frac{3\lambda_{61} v_1^2}{2v_2} \right) \\
&\quad + \frac{1}{4\sqrt{a}} \left( \lambda_1 v_1^2 - \lambda_2 v_2^2 - \frac{m_{12}^{2R}}{v_1 v_2} (v_1^2 - v_2^2) + \frac{3v_1 v_2 (\lambda_{61} - \lambda_{71})}{2} - \frac{\lambda_{71} v_2^3}{2v_1} + \frac{\lambda_{61} v_1^3}{2v_2} \right) \times \\
&\quad \left( \lambda_1 v_1 - \frac{m_{12}^{2R}}{v_2} + (v_1^2 - v_2^2) \frac{m_{12}^{2R}}{2v_1^2 v_2} + \frac{3v_2 (\lambda_{61} - \lambda_{71})}{4} + \frac{\lambda_{71} v_2^3}{4v_1^2} + \frac{3\lambda_{61} v_1^2}{4v_2} \right) \\
&\quad + \frac{1}{2\sqrt{a}} \left( -m_{12}^{2R} + \lambda_{345} v_1 v_2 + \frac{3\lambda_{61} v_1^2}{2} + \frac{3\lambda_{71} v_2^2}{2} \right) \times (\lambda_{345} v_2 + 3\lambda_{61} v_1), \\
F(H, 2) &= \frac{1}{4} \left( 2\lambda_2 v_2 + \frac{m_{12}^{2R}}{v_2^2 v_1} (v_2^2 - v_1^2) + \frac{3\lambda_{67} v_1}{2} - \frac{3\lambda_{71} v_2^2}{2v_1} + \frac{\lambda_{61} v_1^3}{2v_2^2} \right) \\
&\quad + \frac{1}{4\sqrt{a}} \left( \lambda_1 v_1^2 - \lambda_2 v_2^2 - \frac{m_{12}^{2R}}{v_1 v_2} (v_1^2 - v_2^2) + \frac{3v_1 v_2 (\lambda_{61} - \lambda_{71})}{2} - \frac{\lambda_{71} v_2^3}{2v_1} + \frac{\lambda_{61} v_1^3}{2v_2} \right) \times \\
&\quad \left( -\lambda_2 v_2 + \frac{m_{12}^{2R}}{v_1} + (v_1^2 - v_2^2) \frac{m_{12}^{2R}}{2v_2^2 v_1} + \frac{3v_1 (\lambda_{61} - \lambda_{71})}{4} - \frac{3\lambda_{71} v_2^2}{4v_1} - \frac{\lambda_{61} v_1^3}{4v_2^2} \right) \\
&\quad + \frac{1}{2\sqrt{a}} \left( -m_{12}^{2R} + \lambda_{345} v_1 v_2 + \frac{3\lambda_{61} v_1^2}{2} + \frac{3\lambda_{71} v_2^2}{2} \right) \times (\lambda_{345} v_1 + 3\lambda_{71} v_2), \tag{A.2}
\end{aligned}$$

where

$$\begin{aligned}
\lambda_{67} &= \lambda_{61} + \lambda_{71}, \quad \lambda_{345} = \lambda_3 + \lambda_4 + \lambda_{51}, \\
a &= \left( \frac{(\lambda_1 v_1^2 - \lambda_2 v_2^2)}{2} + \frac{3v_1 v_2 (\lambda_{61} - \lambda_{71})}{4} - \frac{\lambda_{71} v_2^3}{4v_1} + \frac{\lambda_{61} v_1^3}{4v_2} - \frac{m_{12}^{2R} (v_1^2 - v_2^2)}{v_1 v_2} \right)^2 \\
&\quad + \left( -m_{12}^{2R} + \lambda_{345} v_1 v_2 + \frac{3\lambda_{61} v_1^2}{2} + \frac{3\lambda_{71} v_2^2}{2} \right)^2. \tag{A.3}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 V_1}{\partial \phi_1^2} \Big|_{\phi_{c1}=v_1} &= \frac{1}{64\pi^2} \left[ 4m_A^2 \left( \frac{m_{12}^{2R} v_2}{v_1^3} - \lambda_{51} - \frac{3\lambda_{61} v_1}{2v_2} + \frac{\lambda_{71} v_2}{2v_1} - \frac{\lambda_{71} v_2 v^2}{2v_1^3} \right) \left( \ln \frac{m_A^2}{\mu^2} - 1 \right) \right. \\
&\quad + 8 \left( \frac{m_{12}^{2R}}{2v_2} \left( 1 - \frac{v_2^2}{v_1^2} \right) - v_1 \lambda_{51} - \frac{\lambda_{61} v^2}{4v_2} - \frac{\lambda_{61} v_1^2}{2v_2} - \frac{\lambda_{71} v_2}{2} + \frac{\lambda_{71} v_2 v^2}{4v_1^2} \right)^2 \ln \frac{m_A^2}{\mu^2} \\
&\quad + 8m_{H^\pm}^2 \left( \frac{m_{12}^{2R} v_2}{v_1^3} - \frac{1}{2} \lambda_{45} - \frac{3\lambda_{61} v_1}{2v_2} + \frac{\lambda_{71} v_2}{2v_1} - \frac{\lambda_{71} v_2 v^2}{2v_1^3} \right) \left( \ln \frac{m_{H^\pm}^2}{\mu^2} - 1 \right) \\
&\quad + 4 \left( \frac{m_{12}^{2R}}{v_2} \left( 1 - \frac{v_2^2}{v_1^2} \right) - v_1 \lambda_{45} - \frac{\lambda_{61} v_1^2}{v_2} - \frac{\lambda_{61} v^2}{2v_2} - \lambda_{71} v_2 + \frac{\lambda_{71} v_2 v^2}{2v_1^2} \right)^2 \ln \frac{m_{H^\pm}^2}{\mu^2} \\
&\quad - 6m_W^2 g_2^2 \left( \frac{1}{3} - \ln \frac{m_W^2}{\mu^2} \right) + g_2^4 v_1^2 \left( 2 + 3 \ln \frac{m_W^2}{\mu^2} \right) - 3m_Z^2 g^2 \left( \frac{1}{3} - \ln \frac{m_Z^2}{\mu^2} \right) \\
&\quad + \frac{g^4 v_1^2}{2} \left( 2 + 3 \ln \frac{m_Z^2}{\mu^2} \right) + 24m_b^2 Y_b^2 \left( 1 - \ln \frac{m_b^2}{\mu^2} \right) - 24v_1^2 Y_b^4 \ln \frac{m_b^2}{\mu^2} \\
&\quad + 8m_\tau^2 Y_\tau^2 \left( 1 - \ln \frac{m_\tau^2}{\mu^2} \right) - 8v_1^2 Y_\tau^4 \ln \frac{m_\tau^2}{\mu^2} + 8 [F(h, 1)]^2 \ln \frac{m_h^2}{\mu^2} \\
&\quad \left. + 4m_h^2 G(h, 1) \left( \ln \frac{m_h^2}{\mu^2} - 1 \right) + 8 [F(H, 1)]^2 \ln \frac{m_H^2}{\mu^2} + 4m_H^2 G(H, 1) \left( \ln \frac{m_H^2}{\mu^2} - 1 \right) \right], \\
\frac{\partial^2 V_1}{\partial \phi_2^2} \Big|_{\phi_{c2}=v_2} &= \frac{1}{64\pi^2} \left[ 4m_A^2 \left( \frac{m_{12}^{2R} v_1}{v_2^3} - \lambda_{51} - \frac{3\lambda_{71} v_2}{2v_1} + \frac{\lambda_{61} v_1}{2v_2} - \frac{\lambda_{61} v_1 v^2}{2v_2^3} \right) \left( \ln \frac{m_A^2}{\mu^2} - 1 \right) \right. \\
&\quad + 8 \left( \frac{m_{12}^{2R}}{2v_1} \left( 1 - \frac{v_1^2}{v_2^2} \right) - v_2 \lambda_{51} - \frac{\lambda_{71} v^2}{4v_1} - \frac{\lambda_{71} v_2^2}{2v_1} - \frac{\lambda_{61} v_1}{2} + \frac{\lambda_{61} v_1 v^2}{4v_2^2} \right)^2 \ln \frac{m_A^2}{\mu^2} \\
&\quad + 8m_{H^\pm}^2 \left( \frac{m_{12}^{2R} v_1}{v_2^3} - \frac{1}{2} \lambda_{45} - \frac{3\lambda_{71} v_2}{2v_1} + \frac{\lambda_{61} v_1}{2v_2} - \frac{\lambda_{61} v_1 v^2}{2v_2^3} \right) \left( \ln \frac{m_{H^\pm}^2}{\mu^2} - 1 \right) \\
&\quad + 4 \left( \frac{m_{12}^{2R}}{v_1} \left( 1 - \frac{v_1^2}{v_2^2} \right) - v_2 \lambda_{45} - \frac{\lambda_{71} v_2^2}{v_1} - \frac{\lambda_{71} v^2}{2v_1} - \lambda_{61} v_1 + \frac{\lambda_{61} v_1 v^2}{2v_2^2} \right)^2 \ln \frac{m_{H^\pm}^2}{\mu^2} \\
&\quad - 6m_W^2 g_2^2 \left( \frac{1}{3} - \ln \frac{m_W^2}{\mu^2} \right) + g_2^4 v_2^2 \left( 2 + 3 \ln \frac{m_W^2}{\mu^2} \right) - 3m_Z^2 g^2 \left( \frac{1}{3} - \ln \frac{m_Z^2}{\mu^2} \right) \\
&\quad + \frac{g^4 v_2^2}{2} \left( 2 + 3 \ln \frac{m_Z^2}{\mu^2} \right) + 24m_t^2 Y_t^2 \left( 1 - \ln \frac{m_t^2}{\mu^2} \right) - 24v_2^2 Y_t^4 \ln \frac{m_t^2}{\mu^2} \\
&\quad + 4m_h^2 G(h, 2) \left( \ln \frac{m_h^2}{\mu^2} - 1 \right) + 8 [F(h, 2)]^2 \ln \frac{m_h^2}{\mu^2} + 4m_H^2 G(H, 2) \left( \ln \frac{m_H^2}{\mu^2} - 1 \right) \\
&\quad \left. + 8 [F(H, 2)]^2 \ln \frac{m_H^2}{\mu^2} \right]. \tag{A.4}
\end{aligned}$$

Here,

$$\begin{aligned}
G(h, 1) = & \frac{1}{4} \left( 2\lambda_1 - \frac{2m_{12}^{2R}}{v_1 v_2} + \frac{2m_{12}^{2R} v^2}{v_1^3 v_2} - \frac{\lambda_{71} v_2^3}{v_1^3} - \frac{3\lambda_{61} v_1}{v_2} \right) \\
& - \frac{1}{4\sqrt{a}} \left[ \left( \lambda_1 v_1^2 - \lambda_2 v_2^2 - \frac{m_{12}^{2R}}{v_1 v_2} (v_1^2 - v_2^2) + \frac{3v_1 v_2 (\lambda_{61} - \lambda_{71})}{2} - \frac{\lambda_{71} v_2^3}{2v_1} + \frac{\lambda_{61} v_1^3}{2v_2} \right) \right. \\
& \times \left( \lambda_1 + \frac{m_{12}^{2R}}{v_1 v_2} - \frac{m_{12}^{2R}}{v_1^3 v_2} (v_1^2 - v_2^2) - \frac{\lambda_{71} v_2^3}{2v_1^3} + \frac{3\lambda_{61} v_1}{2v_2} \right) \\
& + 2 \left( \lambda_1 v_1 - \frac{m_{12}^{2R}}{v_2} + \frac{m_{12}^{2R}}{2v_1^2 v_2} (v_1^2 - v_2^2) + \frac{3v_2}{4} (\lambda_{61} - \lambda_{71}) + \frac{\lambda_{71} v_2^3}{4v_1^2} + \frac{3\lambda_{61} v_1^2}{4v_2} \right)^2 \\
& + 2 (\lambda_{345} v_2 + 3\lambda_{61} v_1)^2 + 6\lambda_{61} \left( -m_{12}^{2R} + \lambda_{345} v_1 v_2 + \frac{3\lambda_{61} v_1^2}{2} + \frac{3\lambda_{71} v_2^2}{2} \right) \\
& + \frac{1}{8a^{3/2}} \left[ \left( \lambda_1 v_1^2 - \lambda_2 v_2^2 - \frac{m_{12}^{2R}}{v_1 v_2} (v_1^2 - v_2^2) + \frac{3v_1 v_2 (\lambda_{61} - \lambda_{71})}{2} - \frac{\lambda_{71} v_2^3}{2v_1} + \frac{\lambda_{61} v_1^3}{2v_2} \right) \right. \\
& \times \left( \lambda_1 v_1 - \frac{m_{12}^{2R}}{v_2} + (v_1^2 - v_2^2) \frac{m_{12}^{2R}}{2v_1^2 v_2} + \frac{3v_2 (\lambda_{61} - \lambda_{71})}{4} + \frac{\lambda_{71} v_2^3}{4v_1^2} + \frac{3\lambda_{61} v_1^2}{4v_2} \right) \\
& \left. + 2 \left( -m_{12}^{2R} + \lambda_{345} v_1 v_2 + \frac{3\lambda_{61} v_1^2}{2} + \frac{3\lambda_{71} v_2^2}{2} \right) \times (\lambda_{345} v_2 + 3\lambda_{61} v_1) \right]^2, \\
& \tag{A.5}
\end{aligned}$$

$$\begin{aligned}
G(h, 2) = & \frac{1}{4} \left( 2\lambda_2 - \frac{2m_{12}^{2R}}{v_1 v_2} + \frac{2m_{12}^{2R} v^2}{v_2^3 v_1} - \frac{3\lambda_{71} v_2}{v_1} - \frac{\lambda_{61} v_1^3}{v_2^3} \right) \\
& - \frac{1}{4\sqrt{a}} \left[ \left( \lambda_1 v_1^2 - \lambda_2 v_2^2 - \frac{m_{12}^{2R}}{v_1 v_2} (v_1^2 - v_2^2) + \frac{3v_1 v_2 (\lambda_{61} - \lambda_{71})}{2} - \frac{\lambda_{71} v_2^3}{2v_1} + \frac{\lambda_{61} v_1^3}{2v_2} \right) \right. \\
& \times \left( -\lambda_2 - \frac{m_{12}^{2R}}{v_1 v_2} - \frac{m_{12}^{2R}}{v_2^3 v_1} (v_1^2 - v_2^2) - \frac{3\lambda_{71} v_2}{2v_1} + \frac{\lambda_{61} v_1^3}{2v_2^3} \right) \\
& + 2 \left( -\lambda_2 v_2 + \frac{m_{12}^{2R}}{v_1} + \frac{m_{12}^{2R}}{2v_2^2 v_1} (v_1^2 - v_2^2) + \frac{3v_1 (\lambda_{61} - \lambda_{71})}{4} - \frac{3\lambda_{71} v_2^2}{4v_1} - \frac{\lambda_{61} v_1^3}{4v_2^2} \right)^2 \\
& + 6\lambda_{71} \left( -m_{12}^{2R} + \lambda_{345} v_1 v_2 + \frac{3\lambda_{61} v_1^2}{2} + \frac{3\lambda_{71} v_2^2}{2} \right) + 2 (\lambda_{345} v_1 + 3\lambda_{71} v_2)^2 \\
& + \frac{1}{8a^{3/2}} \left[ \left( \lambda_1 v_1^2 - \lambda_2 v_2^2 - \frac{m_{12}^{2R}}{v_1 v_2} (v_1^2 - v_2^2) + \frac{3v_1 v_2 (\lambda_{61} - \lambda_{71})}{2} - \frac{\lambda_{71} v_2^3}{2v_1} + \frac{\lambda_{61} v_1^3}{2v_2} \right) \times \right. \\
& \times \left( -\lambda_2 v_2 + \frac{m_{12}^{2R}}{v_1} + (v_1^2 - v_2^2) \frac{m_{12}^{2R}}{2v_2^2 v_1} + \frac{3v_1 (\lambda_{61} - \lambda_{71})}{4} - \frac{3\lambda_{71} v_2^2}{4v_1} - \frac{\lambda_{61} v_1^3}{4v_2^2} \right) \\
& \left. + 2 \left( -m_{12}^{2R} + \lambda_{345} v_1 v_2 + \frac{3\lambda_{61} v_1^2}{2} + \frac{3\lambda_{71} v_2^2}{2} \right) \times (\lambda_{345} v_1 + 3\lambda_{71} v_2) \right]^2, \\
& \tag{A.6}
\end{aligned}$$

$$\begin{aligned}
G(H, 1) = & \frac{1}{4} \left( 2\lambda_1 - \frac{2m_{12}^{2R}}{v_1 v_2} + \frac{2m_{12}^{2R} v^2}{v_1^3 v_2} - \frac{\lambda_{71} v_2^3}{v_1^3} - \frac{3\lambda_{61} v_1}{v_2} \right) \\
& + \frac{1}{4\sqrt{a}} \left[ \left( \lambda_1 v_1^2 - \lambda_2 v_2^2 - \frac{m_{12}^{2R}}{v_1 v_2} (v_1^2 - v_2^2) + \frac{3v_1 v_2 (\lambda_{61} - \lambda_{71})}{2} - \frac{\lambda_{71} v_2^3}{2v_1} + \frac{\lambda_{61} v_1^3}{2v_2} \right) \right. \\
& \times \left( \lambda_1 + \frac{m_{12}^{2R}}{v_1 v_2} - \frac{m_{12}^{2R}}{v_1^3 v_2} (v_1^2 - v_2^2) - \frac{\lambda_{71} v_2^3}{2v_1^3} + \frac{3\lambda_{61} v_1}{2v_2} \right) \\
& + 2 \left( \lambda_1 v_1 - \frac{m_{12}^{2R}}{v_2} + \frac{m_{12}^{2R}}{2v_1^2 v_2} (v_1^2 - v_2^2) + \frac{3v_2}{4} (\lambda_{61} - \lambda_{71}) + \frac{\lambda_{71} v_2^3}{4v_1^2} + \frac{3\lambda_{61} v_1^2}{4v_2} \right)^2 \\
& + 2 (\lambda_{345} v_2 + 3\lambda_{61} v_1)^2 + 6\lambda_{61} \left( -m_{12}^{2R} + \lambda_{345} v_1 v_2 + \frac{3\lambda_{61} v_1^2}{2} + \frac{3\lambda_{71} v_2^2}{2} \right) \left. \right] \\
& - \frac{1}{8a^{3/2}} \left[ \left( \lambda_1 v_1^2 - \lambda_2 v_2^2 - \frac{m_{12}^{2R}}{v_1 v_2} (v_1^2 - v_2^2) + \frac{3v_1 v_2 (\lambda_{61} - \lambda_{71})}{2} - \frac{\lambda_{71} v_2^3}{2v_1} + \frac{\lambda_{61} v_1^3}{2v_2} \right) \right. \\
& \times \left( \lambda_1 v_1 - \frac{m_{12}^{2R}}{v_2} + (v_1^2 - v_2^2) \frac{m_{12}^{2R}}{2v_1^2 v_2} + \frac{3v_2 (\lambda_{61} - \lambda_{71})}{4} + \frac{\lambda_{71} v_2^3}{4v_1^2} + \frac{3\lambda_{61} v_1^2}{4v_2} \right) \\
& + 2 \left( -m_{12}^{2R} + \lambda_{345} v_1 v_2 + \frac{3\lambda_{61} v_1^2}{2} + \frac{3\lambda_{71} v_2^2}{2} \right) \times (\lambda_{345} v_2 + 3\lambda_{61} v_1) \left. \right]^2, \\
\end{aligned} \tag{A.7}$$

$$\begin{aligned}
G(H, 2) = & \frac{1}{4} \left( 2\lambda_2 - \frac{2m_{12}^{2R}}{v_1 v_2} + \frac{2m_{12}^{2R} v^2}{v_2^3 v_1} - \frac{3\lambda_{71} v_2}{v_1} - \frac{\lambda_{61} v_1^3}{v_2^3} \right) \\
& + \frac{1}{4\sqrt{a}} \left[ \left( \lambda_1 v_1^2 - \lambda_2 v_2^2 - \frac{m_{12}^{2R}}{v_1 v_2} (v_1^2 - v_2^2) + \frac{3v_1 v_2 (\lambda_{61} - \lambda_{71})}{2} - \frac{\lambda_{71} v_2^3}{2v_1} + \frac{\lambda_{61} v_1^3}{2v_2} \right) \right. \\
& \times \left( -\lambda_2 - \frac{m_{12}^{2R}}{v_1 v_2} - \frac{m_{12}^{2R}}{v_2^3 v_1} (v_1^2 - v_2^2) - \frac{3\lambda_{71} v_2}{2v_1} + \frac{\lambda_{61} v_1^3}{2v_2^3} \right) \\
& + 2 \left( -\lambda_2 v_2 + \frac{m_{12}^{2R}}{v_1} + \frac{m_{12}^{2R}}{2v_2^2 v_1} (v_1^2 - v_2^2) + \frac{3v_1 (\lambda_{61} - \lambda_{71})}{4} - \frac{3\lambda_{71} v_2^2}{4v_1} - \frac{\lambda_{61} v_1^3}{4v_2^2} \right)^2 \\
& + 6\lambda_{71} \left( -m_{12}^{2R} + \lambda_{345} v_1 v_2 + \frac{3\lambda_{61} v_1^2}{2} + \frac{3\lambda_{71} v_2^2}{2} \right) + 2 (\lambda_{345} v_1 + 3\lambda_{71} v_2)^2 \left. \right] \\
& - \frac{1}{8a^{3/2}} \left[ \left( \lambda_1 v_1^2 - \lambda_2 v_2^2 - \frac{m_{12}^{2R}}{v_1 v_2} (v_1^2 - v_2^2) + \frac{3v_1 v_2 (\lambda_{61} - \lambda_{71})}{2} - \frac{\lambda_{71} v_2^3}{2v_1} + \frac{\lambda_{61} v_1^3}{2v_2} \right) \times \right. \\
& \times \left( -\lambda_2 v_2 + \frac{m_{12}^{2R}}{v_1} + (v_1^2 - v_2^2) \frac{m_{12}^{2R}}{2v_2^2 v_1} + \frac{3v_1 (\lambda_{61} - \lambda_{71})}{4} - \frac{3\lambda_{71} v_2^2}{4v_1} - \frac{\lambda_{61} v_1^3}{4v_2^2} \right) \\
& + 2 \left( -m_{12}^{2R} + \lambda_{345} v_1 v_2 + \frac{3\lambda_{61} v_1^2}{2} + \frac{3\lambda_{71} v_2^2}{2} \right) \times (\lambda_{345} v_1 + 3\lambda_{71} v_2) \left. \right]^2. \\
\end{aligned} \tag{A.8}$$

## References

- [1] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher and J. P. Silva, “Theory and phenomenology of two-Higgs-doublet models,” Phys. Rept. **516**, 1 (2012) [arXiv:1106.0034 [hep-ph]].
- [2] For a recent review, see G. Bhattacharyya and D. Das, “Scalar sector of Two-Higgs-Doublet models: A mini-review,” arXiv:1507.06424 [hep-ph].

- [3] G. C. Branco and M. N. Rebelo, “The Higgs Mass in a Model With Two Scalar Doublets and Spontaneous CP Violation,” *Phys. Lett. B* **160**, 117 (1985).
- [4] G. C. Branco, W. Grimus and L. Lavoura, “Relating the scalar flavor changing neutral couplings to the CKM matrix,” *Phys. Lett. B* **380**, 119 (1996) [hep-ph/9601383].
- [5] F. J. Botella, G. C. Branco, A. Carmona, M. Nebot, L. Pedro and M. N. Rebelo, “Physical Constraints on a Class of Two-Higgs Doublet Models with FCNC at tree level,” arXiv:1401.6147 [hep-ph].
- [6] G. Bhattacharyya, D. Das and A. Kundu, “Feasibility of light scalars in a class of two-Higgs-doublet models and their decay signatures,” *Phys. Rev. D* **89**, 095029 (2014) [arXiv:1402.0364 [hep-ph]].
- [7] S. L. Glashow and S. Weinberg, “Natural Conservation Laws for Neutral Currents,” *Phys. Rev. D* **15**, 1958 (1977).  
E. A. Paschos, “Diagonal Neutral Currents,” *Phys. Rev. D* **15**, 1966 (1977).
- [8] H. S. Cheon and S. K. Kang, “Constraining parameter space in type-II two-Higgs doublet model in light of a 126 GeV Higgs boson,” *JHEP* **1309**, 085 (2013) [arXiv:1207.1083 [hep-ph]].
- [9] B. Coleppa, F. Kling and S. Su, “Constraining Type II 2HDM in Light of LHC Higgs Searches,” *JHEP* **1401**, 161 (2014) [arXiv:1305.0002 [hep-ph]].
- [10] O. Eberhardt, U. Nierste and M. Wiebusch, “Status of the two-Higgs-doublet model of type II,” *JHEP* **1307**, 118 (2013) [arXiv:1305.1649 [hep-ph]].
- [11] O. Eberhardt, “Fitting the Two-Higgs-Doublet model of type II,” arXiv:1405.3181 [hep-ph].
- [12] N. Chakrabarty, U. K. Dey and B. Mukhopadhyaya, “High-scale validity of a two-Higgs doublet scenario: a study including LHC data,” *JHEP* **1412**, 166 (2014) [arXiv:1407.2145 [hep-ph]].
- [13] G. Aad *et al.* [ATLAS and CMS Collaborations], “Combined Measurement of the Higgs Boson Mass in  $pp$  Collisions at  $\sqrt{s} = 7$  and 8 TeV with the ATLAS and CMS Experiments,” *Phys. Rev. Lett.* **114**, 191803 (2015) [arXiv:1503.07589 [hep-ex]].
- [14] D. Chowdhury and O. Eberhardt, “Global fits of the two-loop renormalized Two-Higgs-Doublet model with soft  $Z_2$  breaking,” arXiv:1503.08216 [hep-ph].
- [15] J. Bernon, J. F. Gunion, H. E. Haber, Y. Jiang and S. Kraml, “Scrutinizing the Alignment Limit in Two-Higgs-Doublet Models. Part 1:  $m_h = 125$  GeV,” arXiv:1507.00933 [hep-ph].
- [16] D. Das and I. Saha, “Search for a stable alignment limit in two-Higgs-doublet models,” *Phys. Rev. D* **91**, no. 9, 095024 (2015) [arXiv:1503.02135 [hep-ph]].
- [17] F. Mahmoudi and O. Stal, “Flavor constraints on the two-Higgs-doublet model with general Yukawa couplings,” *Phys. Rev. D* **81**, 035016 (2010) [arXiv:0907.1791 [hep-ph]].
- [18] O. Deschamps, S. Descotes-Genon, S. Monteil, V. Niess, S. T’Jampens and V. Tisserand, “The Two Higgs Doublet of Type II facing flavour physics data,” *Phys. Rev. D* **82**, 073012 (2010) [arXiv:0907.5135 [hep-ph]].
- [19] B. Gorczyca and M. Krawczyk, “Tree-Level Unitarity Constraints for the SM-like 2HDM,” arXiv:1112.5086 [hep-ph].
- [20] A. Barroso, P. M. Ferreira, I. P. Ivanov and R. Santos, “Metastability bounds on the two Higgs doublet model,” *JHEP* **1306**, 045 (2013) [arXiv:1303.5098 [hep-ph]].

- [21] A. Barroso, P. M. Ferreira, I. Ivanov and R. Santos, “Tree-level metastability bounds in two-Higgs doublet models,” arXiv:1305.1235 [hep-ph].
- [22] P. M. Ferreira, R. Santos and A. Barroso, “Stability of the tree-level vacuum in two Higgs doublet models against charge or CP spontaneous violation,” *Phys. Lett. B* **603**, 219 (2004) [*Phys. Lett. B* **629**, 114 (2005)] [hep-ph/0406231].
- [23] I. P. Ivanov, “Minkowski space structure of the Higgs potential in 2HDM,” *Phys. Rev. D* **75**, 035001 (2007) [*Phys. Rev. D* **76**, 039902 (2007)] [hep-ph/0609018].
- [24] I. P. Ivanov, “Minkowski space structure of the Higgs potential in 2HDM. II. Minima, symmetries, and topology,” *Phys. Rev. D* **77**, 015017 (2008) [arXiv:0710.3490 [hep-ph]].
- [25] I. P. Ivanov and J. P. Silva, “Tree-level metastability bounds for the most general two Higgs doublet model,” arXiv:1507.05100 [hep-ph].
- [26] M. Sher, “Electroweak Higgs Potentials and Vacuum Stability,” *Phys. Rept.* **179**, 273 (1989).
- [27] M. Sher, “The Coleman-Weinberg phase transition in extended Higgs models,” *Phys. Rev. D* **54**, 7071 (1996) [hep-ph/9607337].
- [28] J. S. Lee and A. Pilaftsis, “Radiative Corrections to Scalar Masses and Mixing in a Scale Invariant Two Higgs Doublet Model,” *Phys. Rev. D* **86**, 035004 (2012) [arXiv:1201.4891 [hep-ph]].
- [29] S. R. Coleman and E. J. Weinberg, “Radiative Corrections as the Origin of Spontaneous Symmetry Breaking,” *Phys. Rev. D* **7**, 1888 (1973).
- [30] M. Quiros, “Finite temperature field theory and phase transitions,” hep-ph/9901312.
- [31] L. Fromme, S. J. Huber and M. Seniuch, *JHEP* **0611**, 038 (2006) [hep-ph/0605242]; A. Tranberg and B. Wu, *JHEP* **1301**, 046 (2013) [arXiv:1210.1779 [hep-ph]].
- [32] N. Khan and S. Rakshit, “Constraints on inert dark matter from metastability of electroweak vacuum,” arXiv:1503.03085 [hep-ph].
- [33] B. Swiezewska, “Inert scalars and vacuum metastability around the electroweak scale,” *JHEP* **1507**, 118 (2015) [arXiv:1503.07078 [hep-ph]].
- [34] E. Gildener and S. Weinberg, “Symmetry Breaking and Scalar Bosons,” *Phys. Rev. D* **13**, 3333 (1976).
- [35] S. P. Martin, “Taming the Goldstone contributions to the effective potential,” *Phys. Rev. D* **90**, no. 1, 016013 (2014) [arXiv:1406.2355 [hep-ph]].
- [36] I. Chakraborty and A. Kundu, “Two-Higgs doublet models confront the naturalness problem,” *Phys. Rev. D* **90**, no. 11, 115017 (2014) [arXiv:1404.3038 [hep-ph]].